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GENERAL SOLUTION OF THE LAMINAR COMPRESSIBLE BOUNDARY LAYER IN THE STAGNATION REGION OF BLUNT BODIES IN AXISYMMETRIC FLOW

by Fred W. Matting

Ames Research Center
Moffett Field, Calif.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION • WASHINGTON, D. C. • JULY 1964

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NOMENCLATURE

a_i	coefficients in expansion of $u_e/\sqrt{2H_{st}}$ (eq. (3.4))
b_i	coefficients in expansion of $u_e^2/2H_{st}$ (eqs. (3.6), (3.7))
c_p	specific heat at constant pressure, ft-lb/slug ^oR = ft^2/sec^2 ^oR
C_{fe}	coefficient of local skin friction based on q_e, $\bar{\tau}_w/q_e$, dimensionless
C_{fw}	coefficient of local skin friction based on q_w, $\bar{\tau}_w/q_w$, dimensionless
D	diffusion coefficient (binary), ft^2/sec
f	dimensionless stream function (eq. (2.15))
$\partial f/\partial\eta$	dimensionless longitudinal velocity (eq. (2.17)), u/u_e
f_j	jth order term in expansion of f in powers of ξ^j (eq. (3.2))
${}_if_j$	split form of f_j (see eq. (A5b))
f_o	zero order of f_j (order ξ^o) = value of $f(\eta)$ at the stagnation point
f_o'	zero order (ξ^o) of dimensionless longitudinal velocity, u/u_e (at the stagnation point), $df_o/d\eta$
F	dimensionless stream function; $f_o = f(\eta)$ at the stagnation point (eq. (4.1a))
F'	dimensionless velocity; $f_o'(\eta) = u/u_e$ at the stagnation point
F_j	jth order term in expansion of F in powers of $(hwo - 1)^j$ (eq. (4.7))
${}_iF_j$	split form of F_j (appendix B and eq. (5.4))
g	$\rho\mu/(\rho\mu)_s$ (eq. (2.21a))
g_i	coefficients in expansion of g (eqs. (3.1a), (5.7b))
G	dimensionless enthalpy; $h_o = h(\eta)$ at the stagnation point (eq. (4.1b))
G_j	jth order term in expansion of G in powers of $(hwo - 1)^j$ (eq. (4.6))
${}_iG_j$	split form of G_j (appendix B and eq. (5.3))
h	dimensionless enthalpy, H/H_{st}
h_j	jth order term in expansion of h in powers of ξ^j (eq. (3.3))
${}_ih_j$	split form of h_j (see eq. (A5a))
h_o	zero order of h_j (order ξ^o); value of $h(\eta)$ at the stagnation point

hw dimensionless wall enthalpy, H_w/H_{st} (exception to w subscript notation)

hwi ith order of expansion of hw in powers of ξ^i (eq. (3.9))

hwo zero order of hwi (order ξ^0); that is, the stagnation point value of hw

H enthalpy, ft-lb/slug = ft^2/sec^2

H_{st} stagnation enthalpy of exterior adiabatic flow measured at the stagnation point, ft-lb/slug = ft^2/sec^2

${}_iI_j$ integral of enthalpy function, $\int_0^\eta {}_iG_j \, d\eta_1$, dimensionless (eq. (5.1))

k coefficient of thermal conductivity, ft-lb/ft sec ^oR

Le Lewis number, $\rho D c_p/k$, dimensionless

m $(\rho\mu/\sigma)/(\rho\mu/\sigma)_s$ (eq. (2.28))

m_i coefficients in expansion of m (eqs. (3.1b), (5.7a))

M Mach number

M_∞ free-stream or flight Mach number

Ng highest order term in g polynomial (eq. (3.1a))

Nm highest order term in m polynomial (eq. (3.1b))

Nδ highest order term in δ polynomial (eq. (3.1c))

Nu_w local Nusselt number based on wall properties, dimensionless (eq. (6.48))

P pressure, lb/ft^2

P_{t2} stagnation pressure behind normal shock, lb/ft^2

q_e dynamic pressure based on ρ_e, $(1/2)\rho_e u_e^2$, lb/ft^2

q_w dynamic pressure based on ρ_w, $(1/2)\rho_w u_e^2$, lb/ft^2

Q rate of heat transfer per unit area by conduction in the y direction, ft-lb/sec ft^2

r body radius from axis, ft

R gas constant for thermally perfect gases, $P/\rho T$, ft-lb/^oR slug = $ft^2/{}^oR$ sec^2

Re Reynolds number based on gas properties at outer edge of boundary layer, $u_e\rho_e x/\mu_e$, dimensionless

Rw Reynolds number based on gas properties at the wall, $u_e\rho_w x/\mu_w$, dimensionless

St_w local Stanton number based on wall properties, dimensionless (eq. (6.52))

T temperature, ^oR

u longitudinal velocity component, ft/sec

u_e longitudinal velocity at outer edge of boundary layer, ft/sec

v transverse velocity component, ft/sec

x longitudinal coordinate along meridian of body, ft

y transverse coordinate, normal to surface of body, ft

Z compressibility function of gases, $P/\rho RT$, dimensionless

δ ρ_s/ρ (eq. (2.24a)

δ_i coefficients in expansion of δ (eqs. (3.1c), (5.7c))

δ^* displacement thickness, ft (eq. (6.20))

Δ value of η at outer edge of boundary layer, dimensionless

ϵ exponent: 0 for two-dimensional flows, 1 for axisymmetric flows (eq. (2.1))

η transformed transverse coordinate, dimensionless (eq. (2.8)); specializes to equation (2.40) for axisymmetric flow

μ viscosity, lb-sec/ft^2

ξ transformed longitudinal coordinate (final transformation); for axisymmetric flow (lb-sec/ft)$^{1/2}$ (eq. (2.39))

ρ density, slugs/ft^3 = lb-sec^2/ft^4

σ Prandtl number, $c_p\mu/k$, dimensionless

σ_o reference Prandtl number

σ_1 defined by equation (4.2)

τ transformed longitudinal coordinate (intermediate transformation); for axisymmetric flow (lb^2-sec^2/ft^2) (eq. (2.7))

$\bar{\tau}$ shear stress, lb/ft^2

ψ stream function; for axisymmetric flow, slugs/sec = lb-sec/ft

Subscripts

AP quantity determined using an "almost perfect" gas, defined as having a linear relationship between δ and G (or h) (eq. (6.9))

aw adiabatic wall

e	outer edge of boundary layer
eff	effective (see ch. 7, B.1.a)
i,j,n,o, 2,4,...	summation indexes and identification, for example, ${}_iF_j$, ${}_iG_j$; the right-side index (j) refers to an order in powers of ξ^j or $(hwo - 1)^j$
P	quantity determined using a "perfect" gas (calorically and thermally perfect), defined as "almost perfect" gas (AP), with $\delta_1 = 1$ (eq. (6.10))
s	a gas property at the stagnation enthalpy of the exterior adiabatic flow, H_{st}, and at the pressure of the ξ or x station; for example, $(\rho\mu)_s = (\rho\mu)[H_{st}, P(\xi)]$
si	ith order of expansion in powers of ξ^i of quantities with subscript s
so	zero order of si (order ξ^o), that is, the stagnation point value; for example, $(\rho\mu)_{so} = (\rho\mu)[H_{st}, P(0)]$
st	stagnation (used for H_{st} only)
w	wall value of a quantity, or a gas property measured at the wall enthalpy, H_w, and at the pressure of the ξ, (x) station; for example, $\rho_w = \rho[H_w(\xi), P(\xi)]$
wi	ith order of expansion in powers of ξ^i of quantities with subscript w
wo	zero order of wi (order ξ^o), that is, the wall value at the stagnation point; for example, $g_{wo} = g(hwo)$
x,y,η,ξ,τ	partial differentiation
1	dummy variable of integration

Superscript

'	differentiation with respect to η

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SUMMARY

A modified Howarth-Dorodnitsyn transformation has been used to write the laminar boundary-layer equations for axisymmetric flow in terms of transformed variables, and the equations have been specialized for the vicinity of the stagnation point. Solutions to the equations depend on the wall enthalpy and on the variation of several of the gas properties through the boundary layer. The gas-property relationships are represented in the form of polynomials in powers of the deviation of the dimensionless enthalpy from a reference value, and the coefficients of these polynomials become parameters of the problem.

The solution of the stagnation-point equations is obtained by expanding the dimensionless enthalpy and stream function into two series in powers of the deviation of the dimensionless wall enthalpy from a reference value. With this formulation the effects of all the parameters appear explicitly. The equations obtained by this procedure determine the universal functions of the problem.

Values of the universal functions are tabulated and working formulas are compiled including inverse transformations back to field variables. Examples are given to demonstrate the use of the universal functions. Comparisons are made with exact solutions for velocity and enthalpy profiles, for wall-friction coefficients, and for heat transfer at the wall.

The solutions obtained are applicable to any gas. They are best suited for free-stream Mach numbers, up to about 5, where chemical reactions are not especially important. At the higher Mach numbers with chemical reactions, the calculations are for equilibrium-gas boundary layers.

*This report was submitted to Stanford University in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Engineering Mechanics, June 1964.

CHAPTER 1

INTRODUCTION

The boundary-layer equations proposed by Prandtl in 1904 and extended to compressible flow by the inclusion of an energy equation (ref. 1) are generally used for solving compressible laminar-boundary-layer problems although they represent only a first approximation of the Navier-Stokes equations. In Prandtl's treatment the flow field has been separated into two parts, an external inviscid flow and a boundary layer. The usual procedure is to solve the external flow problem first, applying a tangency flow boundary condition at the actual body shape. This neglects the displacement effect on the external flow due to the boundary layer on the body.

In the solution of the external flow problem, vorticity in the external flow is often neglected. Considering vorticity in the external flow changes (and complicates) the outer boundary condition for the boundary-layer equations. Around bodies whose longitudinal curvature is not large, the centrifugal force term in the boundary-layer momentum equations is generally neglected, allowing the momentum equation normal to the body surface to be discarded.

It is seen that if one considers the displacement of the external flow by the boundary layer and considers also the external vorticity, there is a complicated interaction between the boundary layer and the external flow. One approach to this interaction problem is to numerically solve the boundary-layer equations over the flow field. Flügge-Lotz and coworkers have obtained a number of solutions with interaction effects by numerical methods. In reference 2, for example, the displacement-thickness interaction has been treated using an implicit finite-difference scheme, and in reference 3, all interactions have been considered using finite-difference solutions to equations developed from inner and outer expansions by Van Dyke (refs. 4 and 5).

The consideration of interactions in the boundary-layer problem is outside the scope of the present work. The centrifugal force term will be neglected by the assumption that the curvature around a blunt body is not large. For the inner boundary condition, a no-slip, no-temperature jump, and no-injection condition will be assumed.

Solutions will be considered in the nature of a first-order approximation to the actual flow as described by the full Navier-Stokes and energy equations.

The compressible laminar boundary-layer problem, even without interactions, is complicated because the differential equations are coupled and nonlinear, and, in general, do not have closed-form solutions. The problem can be solved by numerical (finite difference) methods, and, generally, one must obtain a new solution for each "case." The blunt-body problem can be generalized somewhat if the boundary-layer equations are expanded into powers of the longitudinal variable. Then, analogously to the Blasius series for incompressible flow (ref. 1), the problem is reduced to the solution of a number of ordinary differential equations, the zero-order equations being nonlinear, the higher-order equations linear, but all equations essentially coupled. The solutions for these ordinary differential equations are not universal for compressible flow, because they depend on the wall temperature (or enthalpy) and on the variation of the thermodynamic and transport properties of the gas.

In the present work we will set up the boundary-layer equations in transformed variables (ch. 2), using a modified form of the Howarth-Dorodnitsyn transformation (ref. 6). We will specialize the transformed equations for the vicinity of the stagnation point of axisymmetric blunt bodies in axisymmetric flow (ch. 3). The resulting equations will be a pair of coupled nonlinear ordinary differential equations. These will be the zero-order equations in the expansion of the transformed partial differential equations in powers of the transformed longitudinal variable. (The forms of the higher-order equations in this expansion are indicated in appendix A.)

We will seek a general form of solutions to the stagnation-point equations. These solutions will be made up of sums of the products of universal functions and the parameters of the problem, and they will depend on the wall enthalpy and on the variation of several of the gas properties through the boundary layer. The gas properties can be related to the enthalpy of the gas, either analytically or numerically. We will represent the functional relationship between the several gas

properties and the enthalpy in the form of polynomials in powers of the deviation of the dimensionless enthalpy from a reference value. The coefficients of the polynomials become parameters of the problem; the universal functions are independent of the particular values that the parameters may take.

Our method of solving the stagnation-point equations is to form the solution as a series in powers of the deviation of the dimensionless wall enthalpy from a reference value. With this formulation, the effects of all the other parameters appear explicitly. The equations are set up in the form to be solved in chapter 4. The obtaining of the solutions for the universal functions is explained in chapter 5; these solutions were all obtained numerically by machine computation. Tabulations of these functions are given (table I); they are also stored on a permanent binary tape for convenient use in machine computation* although an automatic computing machine is not needed for the use of the functions. Inverse transformations back to field variables, and the determination of a number of coefficients of interest are given in chapter 6. Examples explaining the use of the method of solution are given in chapter 7.

The range of validity of the calculations is discussed in chapter 8. In general, the procedure, as set up, is best suited to moderate free-stream Mach numbers, say up to about 5, where chemical effects are not especially important. At the higher Mach numbers, with chemical reactions occurring in the boundary layer, the procedure will deliver a reasonable approximation to actual flows. The calculation is for an equilibrium boundary layer and does not consider finite chemical reaction rates. This is not inherent in the method of solution, but the original boundary-layer equations from which the solution is developed have been written in a simplified form that does not consider finite chemical reaction rates.

A synopsis of the method of using the universal functions is given in a summary of working formulas in table E-1.

*Available with computing program from Ames Research Center, NASA, Moffett Field, Calif.

CHAPTER 2

PARTIAL DIFFERENTIAL EQUATIONS FOR THE COMPRESSIBLE LAMINAR BOUNDARY LAYER

A. A SYSTEM OF NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

The Navier-Stokes equations as simplified by Prandtl's order of magnitude analysis may be put in the familiar form of the steady-state laminar boundary-layer equations as follows. (The x and y subscripts denote partial differentiation with respect to that variable.)

Continuity equation:

$$(\rho u r^{\epsilon})_x + (\rho v r^{\epsilon})_y = 0 \tag{2.1}$$

x momentum equation:

$$\rho u u_x + \rho v u_y = -P_x + (\mu u_y)_y \tag{2.2}$$

y momentum equation:

$$P_y = 0 \tag{2.3}$$

Energy equation:

$$\rho u H_x + \rho v H_y = u P_x + \mu (u_y)^2 + \left(\frac{\mu}{\sigma} H_y\right)_y \tag{2.4}$$

where

$\epsilon = 0$ for two-dimensional flow

$\epsilon = 1$ for axially symmetric flow

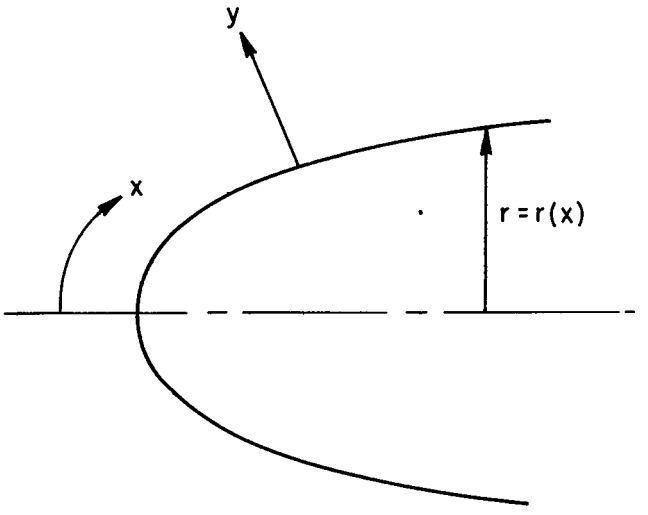
The momentum equation in the y direction, $P_y = 0$, includes the condition that the wall curvature is not strong so that centrifugal force can be neglected. The equation states that the pressure can be assumed constant across the boundary layer. At a distance from the wall at which viscous effects are no longer important, the longitudinal component of velocity, u, approaches asymptotically the external velocity, u_e. At this distance, the longitudinal momentum equation reduces to:

$$P_x = -\rho_e u_e u_{e_x} \tag{2.5}$$

which is the Euler momentum equation for inviscid flow. This equation neglects the contribution to the external pressure due to the transverse velocity component, v_e.

Equations (2.1) to (2.4) are continuum equations and do not consider nonequilibrium chemical effects. The equations can be considered to include diffusion for the equilibrium case (equilibrium gas properties) when σ is an "effective" Prandtl number. (This is explained in ch. 7.) When equilibrium gas properties are used the equations will also give wall values that reasonably approximate "frozen flow" with a catalytic wall (ch. 7). In setting up the boundary conditions for equations (2.1) to (2.4), we will not consider the interaction effects due to external vorticity and displacement thickness. With a tangency flow condition applied on the body surface, the external flow is assumed to be known and the pressure, $P(x)$, throughout the boundary layer is known.

The x coordinate is measured along the body (a meridian in the axisymmetric case), while the y coordinate is measured along an outward normal from the body. Again it is noted that the body curvature should not be strong to permit the use of equations (2.1) to (2.4) with a "bent" coordinate system. The body radius, r, is as shown in the sketch.



Equations (2.1), (2.2), and (2.4) contain six unknowns: u, v, H, ρ, μ, σ. For a given gas, an equation of state, $\rho = \rho(H,P)$, will be known and the gas properties, $\mu = \mu(H,P)$ and $\sigma = \sigma(H,P)$, will be known (ref. 10). The unknowns in the three differential equations are then reduced to three, u, v, H.

The wall boundary conditions with no slip, no injection or suction, and no temperature jump are:

$$u(x,0) = 0 \qquad v(x,0) = 0 \qquad H(x,0) = H_w(x)$$

where $H_w(x)$ is prescribed, or:

$$\frac{\partial H}{\partial y}(x,0) = \text{a prescribed function of } x$$

The external boundary conditions are:

$$u(x,\infty) = u_e(x) \qquad H(x,\infty) = H_e(x)$$

For the enthalpy boundary condition, $H_e(x)$ is determined from the adiabatic relationship:

$$H_e(x) = H_{st} - \frac{u_e^2(x)}{2} \tag{2.6}$$

which neglects the contribution of the transverse velocity component, v_e. For the general problem a starting profile at a given x value should be known to begin the solution. In the case of the symmetrical stagnation-point problem, the partial differential equations can be reduced to ordinary differential equations and the stagnation profile determined. (This will be taken up subsequently.)

Equations (2.1), (2.2), and (2.4) will be transformed by means of the Howarth-Dorodnitsyn transformation (ref. 6, p. 290 of ref. 7), and a stream function will be introduced that satisfies the continuity equation and eliminates u and v. The transformed equations in final form will be of third order in the transformed y coordinate but the enthalpy term will remain of second order in the transformed y.

B. HOWARTH-DORODNITSYN TRANSFORMATION OF THE PARTIAL DIFFERENTIAL EQUATIONS

The following transformation of the coordinates will be used to transform equations (2.1), (2.2), and (2.4):

$$\tau = \int_0^x \rho_w \mu_w u_e r^{2\epsilon} \, dx_1 \tag{2.7}$$

$$\eta = \frac{r^{\epsilon} u_e}{\sqrt{2\tau}} \int_0^y \rho \, dy_1 \tag{2.8}$$

Then:

$$\frac{\partial}{\partial x} = \tau_x \frac{\partial}{\partial \tau} + \eta_x \frac{\partial}{\partial \eta} \qquad \text{and} \qquad \frac{\partial}{\partial y} = \eta_y \frac{\partial}{\partial \eta}$$

because $\tau = \tau(x)$, while $\eta = \eta(x,y)$.

1. Continuity Relationships

The continuity equation (2.1) is transformed to:

$$(\rho u r^{\epsilon})_{\tau}\tau_x + (\rho u r^{\epsilon})_{\eta}\eta_x + r^{\epsilon}(\rho v)_{\eta}\eta_y = 0 \tag{2.9}$$

The original equation, (2.1), can be identically satisfied by a stream function $\psi(x,y)$ such that

$$\rho u = \frac{1}{r^{\epsilon}} \frac{\partial \psi}{\partial y} \tag{2.10}$$

$$\rho v = \frac{-1}{r^{\epsilon}} \frac{\partial \psi}{\partial x} \tag{2.11}$$

Then, by means of the coordinate transformation, we obtain

$$\rho u = \frac{1}{r^{\epsilon}} \frac{\partial \psi(\tau,\eta)\eta_y}{\partial \eta}$$

$$\rho u = \frac{\rho u_e}{\sqrt{2\tau}} \frac{\partial \psi(\tau,\eta)}{\partial \eta} \tag{2.12}$$

$$u = \frac{u_e}{\sqrt{2\tau}} \frac{\partial \psi(\tau,\eta)}{\partial \eta} \tag{2.13}$$

$$\rho v = \frac{-1}{r^{\epsilon}} \left(\frac{\partial \psi}{\partial \tau} \tau_x + \frac{\partial \psi}{\partial \eta} \eta_x \right) \tag{2.14}$$

It is convenient to introduce

$$f(\tau,\eta) = \frac{\psi(\tau,\eta)}{\sqrt{2\tau}} \tag{2.15}$$

The quantity, f, is a kind of dimensionless stream function; it will be a dependent variable in the subsequent equations. The velocity components are then given by

$$u = u_e f_{\eta} = u_e \frac{f_y}{\eta_y} \tag{2.16}$$

or

$$\frac{u}{u_e} = f_{\eta} \tag{2.17}$$

and

$$\rho v = r^{-\epsilon}\left[\left(\sqrt{2\tau}\, f_\tau + \frac{f}{\sqrt{2\tau}}\right)\tau_x + \sqrt{2\tau}\, f_\eta \eta_x\right] \tag{2.18}$$

Equation (2.18) can also be obtained when (2.16) is substituted in (2.9) and formally integrated after some manipulation. With u and v taken as zero at the wall and $u = u_e$ far away from the wall, equations (2.17) and (2.18) provide the boundary conditions on f.

$$f(\tau,0) = 0$$

$$f_\eta(\tau,0) = 0$$

$$f_\eta(\tau,\infty) = 1$$

The relationship in equation (2.18) will be used to eliminate v from the momentum and energy equations.

2. Transformation of the Momentum Equation

We write the momentum equation in the new coordinate system.

$$\rho u(u_\tau \tau_x + u_\eta \eta_x) + \rho v u_\eta \eta_y = \rho_e u_e u_{e_\tau} \tau_x + (\mu u_\eta \eta_y)_\eta \eta_y \tag{2.19}$$

Then we substitute (2.17) and (2.18) in (2.19)

$$\rho u_e f_\eta[(u_{e_\tau} f_\eta + u_e f_{\tau\eta})\tau_x + u_e f_{\eta\eta}\eta_x]$$

$$- \frac{u_e f_{\eta\eta}\eta_y}{r^\epsilon}\left[\left(\sqrt{2\tau}\, f_\tau + \frac{f}{\sqrt{2\tau}}\right)\tau_x + \sqrt{2\tau}\, f_\eta \eta_x\right] = \rho_e u_e u_{e_\tau}\tau_x + (\mu u_e f_{\eta\eta}\eta_y)_\eta \eta_y$$

Substituting τ_x, η_x, and η_y from (2.7) and (2.8) and modifying, we obtain

$$\left[\left(\frac{\rho\mu}{\rho_w\mu_w}\right) f_{\eta\eta}\right]_\eta + f f_{\eta\eta} + 2\left(\frac{d\ ln\ u_e}{d\ ln\ \tau}\right)\left(\frac{\rho_e}{\rho} - f_\eta^2\right) = 2\tau(f_\eta f_{\tau\eta} - f_\tau f_{\eta\eta}) \tag{2.20}$$

We can set

$$\frac{\rho\mu}{\rho_w\mu_w} = \frac{\rho\mu/(\rho\mu)_s}{(\rho_w\mu_w)/(\rho\mu)_s}$$

where $(\rho\mu)_s$ is the product $\rho\mu$ evaluated at the stagnation enthalpy, H_{st}, and at the pressure at the particular τ (or x) station of the boundary layer. In addition we introduce the notation

$$g = \frac{\rho\mu}{(\rho\mu)_s} \tag{2.21a}$$

$$g_w = \frac{\rho_w\mu_w}{(\rho\mu)_s} \tag{2.21b}$$

The type of normalization used for $\rho\mu$ is convenient because, even for real gases, g will be a slowly changing function of the pressure P, and for perfect gases it will not depend on P at all (see appendix A, sect. C). If we normalize the enthalpy by

$$h = \frac{H}{H_{st}} \tag{2.22}$$

we can write

$$g = g(h;\ H_{st}, P)$$

$$g_w = g(hw;\ H_{st}, P)$$

where hw is the dimensionless wall enthalpy at the τ (or x) station considered. For a given flow, H_{st} will be a constant determined by the adiabatic external flow.

After introduction of g the first term of equation (2.20) becomes

$$\left(\frac{g}{g_w} f_{\eta\eta}\right)_\eta = \frac{1}{g_w}\left(\frac{\partial g}{\partial \eta} f_{\eta\eta} + g f_{\eta\eta\eta}\right)$$

with

$$\frac{\partial g}{\partial \eta} = \left(\frac{\partial g}{\partial h}\right) h_\eta + \left(\frac{\partial g}{\partial P}\right) P_\eta$$

but

$$P_\eta = 0$$

So we have

$$\left(\frac{g}{g_w} f_{\eta\eta}\right)_\eta = \frac{1}{g_w}\left[\left(\frac{\partial g}{\partial h}\right) h_\eta f_{\eta\eta} + g f_{\eta\eta\eta}\right] \tag{2.23}$$

The ratio ρ_e/ρ will be treated in a similar way; we set

$$\frac{\rho_e}{\rho} = \frac{\rho_s \rho_e}{\rho_s \rho} = \frac{\rho_s/\rho}{\rho_s/\rho_e}$$

where ρ_s is the density at the stagnation enthalpy, H_{st}, and at the pressure at the particular τ (or x) station. We set

$$\frac{\rho_s}{\rho} = \delta(h;\ H_{st}, P) \tag{2.24a}$$

For perfect gases, δ does not depend on P, and for real gases it varies only slowly with P (see appendix A, sect. C). With

$$\delta_e = \delta(h_e;\ H_{st}, P) = \frac{\rho_s}{\rho_e} \tag{2.24b}$$

where

$$h_e = \frac{H_e}{H_{st}} = 1 - \frac{u_e^2}{2H_{st}} \tag{2.25}$$

we obtain

$$\frac{\rho_e}{\rho} = \frac{\delta}{\delta_e}$$

Then we can write the momentum equation in the following form:

$$\frac{1}{g_w}\left[\left(\frac{\partial g}{\partial h}\right) h_\eta f_{\eta\eta} + g f_{\eta\eta\eta}\right] + f f_{\eta\eta} + 2\left(\frac{d\ ln\ u_e}{d\ ln\ \tau}\right)\left(\frac{\delta}{\delta_e} - f_\eta^2\right)$$

$$= 2\tau\left(f_\eta f_{\tau\eta} - f_\tau f_{\eta\eta}\right) \tag{2.26}$$

3. Transformation of the Energy Equation

In transforming the energy equation (2.4), we first write it in terms of h.

$$\rho u h_x + \rho v h_y = \frac{u P_x}{H_{st}} + \frac{\mu}{H_{st}}(u_y)^2 + \left(\frac{\mu}{\sigma} h_y\right)_y \tag{2.27}$$

Then we introduce the dimensionless stream function, f.

$$\rho u_e f_\eta h_x + (\rho v) h_y = - \frac{\rho_e u_e^2 u_{e_x} f_\eta}{H_{st}} + \mu \left(\frac{u_e^2}{H_{st}}\right) (f_{\eta y})^2 + \left(\frac{\mu}{\sigma} h_y\right)_y$$

We now transform to the variables, τ, η, and substitute ρv from equation (2.18).

$$\rho u_e f_\eta (h_\tau \tau_x + h_\eta \eta_x) - \frac{h_\eta \eta_y}{r^\epsilon} \left[\left(\sqrt{2\tau}\, f_\tau + \frac{f}{\sqrt{2\tau}}\right) \tau_x + \sqrt{2\tau}\, f_{\eta\eta} x\right]$$

$$= - \frac{\rho_e u_e^2 u_{e_\tau} \tau_x f_\eta}{H_{st}} + \mu \left(\frac{u_e^2}{H_{st}}\right) (\eta_y)^2 (f_{\eta\eta})^2 + \left(\frac{\mu}{\sigma} \eta_y h_\eta\right)_\eta \eta_y$$

We substitute the value of τ_x (eq. (2.7)) and η_y (eq. (2.8)) and rearrange

$$2\tau(f_\eta h_\tau - h_\eta f_\tau) - f h_\eta = - \frac{2\tau \rho_e u_e u_{e_\tau} f_\eta}{\rho H_{st}}$$

$$+ \left(\frac{\rho\mu}{\rho_w \mu_w}\right) \left(\frac{u_e^2}{H_{st}}\right) (f_{\eta\eta})^2 + \frac{1}{\sigma_w} \left(\frac{\sigma_w \rho \mu}{\sigma \rho_w \mu_w} h_\eta\right)_\eta$$

The last term contains $\sigma_w \rho\mu / \sigma \rho_w \mu_w$ which can be treated like $\rho\mu$ by setting

$$\frac{\sigma_w \rho\mu}{\sigma \rho_w \mu_w} = \frac{\rho\mu/\sigma}{\rho_w \mu_w / \sigma_w} = \frac{(\rho\mu/\sigma)/(\rho\mu/\sigma)_s}{[(\rho_w \mu_w)/\sigma_w]/(\rho\mu/\sigma)_s}$$

where $(\rho\mu/\sigma)_s = \rho\mu/\sigma$ at the stagnation enthalpy and at the pressure at the particular x station of the boundary layer. We introduce the function

$$m = \frac{\rho\mu/\sigma}{(\rho\mu/\sigma)_s} \tag{2.28}$$

As in the case with g

$$m = m(h;\ H_{st}, P) \tag{2.29a}$$

$$m_w = m(hw;\ H_{st}, P) \tag{2.29b}$$

The quantity, m, is normalized in the same way as g. For perfect gases we can say that m does not depend on the pressure, and for real gases it varies only slowly with pressure (see appendix A, sect. C).

We can now write the energy equation as

$$\frac{1}{\sigma_w m_w}(mh_\eta)_\eta + fh_\eta + 2\frac{g}{g_w}\left(\frac{u_e^2}{2H_{st}}\right)(f_{\eta\eta})^2$$

$$- 2\tau\left[f_\eta h_\tau - h_\eta f_\tau + \left(\frac{\rho_e}{\rho}\right)\frac{d}{d\tau}\left(\frac{u_e^2}{2H_{st}}\right)f_\eta\right] = 0$$

We evaluate $\partial m/\partial\eta$ as we treated $\partial g/\partial\eta$, using the relation $P_\eta = 0$, and we get

$$\frac{\partial m}{\partial \eta} = \left(\frac{\partial m}{\partial h}\right)h_\eta$$

Then we have

$$g_w\left[\left(\frac{\partial m}{\partial h}\right)h_\eta^2 + mh_{\eta\eta}\right] + \sigma_w m_w g_w f h_\eta + 2\sigma_w m_w g\left(\frac{u_e^2}{2H_{st}}\right)f_{\eta\eta}^2$$

$$-2\tau\sigma_w m_w g_w\left[f_\eta h_\tau - h_\eta f_\tau + \left(\frac{\delta}{\delta_e}\right)\frac{d}{d\tau}\left(\frac{u_e^2}{2H_{st}}\right)f_\eta\right] = 0 \qquad (2.30)$$

The energy equation can be used in the form (eq. (2.30)) above. However, the term δ/δ_e is somewhat inconvenient but it can be removed if we solve the momentum equation (2.26) for this term and substitute back in the energy equation. This is done at the cost of making the energy equation somewhat longer than in the form above (eq. (2.30)). With this substitution we obtain

$$g_w\left[\left(\frac{\partial m}{\partial h}\right)h_\eta^2 + mh_{\eta\eta}\right] + \sigma_w m_w g_w f h_\eta$$

$$+ 2\sigma_w m_w\left(\frac{u_e^2}{2H_{st}}\right)\left[\left(\frac{\partial g}{\partial h}\right)h_\eta f_\eta f_{\eta\eta} + g(f_{\eta\eta}^2 + f_\eta f_{\eta\eta\eta}) + g_w f f_\eta f_{\eta\eta}\right]$$

$$- 2\tau\sigma_w m_w g_w\left[f_\eta h_\tau - h_\eta f_\tau + 2\left(\frac{u_e^2}{2H_{st}}\right)(f_\eta^2 f_{\tau\eta} - f_\tau f_\eta f_{\eta\eta}) + \frac{d}{d\tau}\left(\frac{u_e^2}{2H_{st}}\right)f_\eta^3\right] = 0 \qquad (2.31)$$

We will use the energy equation in the form (2.31).

4. Boundary Conditions

Equations (2.26) and (2.31) are a pair of coupled nonlinear partial differential equations in the dependent variables, f and h. The boundary conditions at the wall are

$$f(\tau,0) = f_\eta(\tau,0) = 0 \tag{2.32a}$$

$$h(\tau,0) = hw(\tau) \quad \text{or} \quad h_\eta(\tau,0) = h_{\eta w}(\tau) \tag{2.32b}$$

and for the transition into the exterior flow

$$f_\eta(\tau,\infty) = 1 \tag{2.32c}$$

$$h(\tau,\infty) = h_e(\tau) = 1 - \frac{u_e^2}{2H_{st}} \tag{2.32d}$$

C. FURTHER TRANSFORMATION OF THE PARTIAL DIFFERENTIAL EQUATIONS

The momentum equation (2.26) and the energy equation (2.31) as given above are valid for both two-dimensional and axisymmetric flows. In order to solve equations (2.26) and (2.31), the body shape and the external flow must be known (can be numerical): $r = r(x)$ and $u_e = u_e(x)$. The transformation from x to τ gives: $u_e = u_e(\tau)$ (can be numerical). We can expand the momentum and energy equations in series in powers of the longitudinal variable. This series representing the flow from the stagnation point will involve fractional powers of the variable τ (ref. 7, p. 322). To avoid fractional powers we can make a second transformation to a new longitudinal variable, ξ. The second transformation will result in a separation of the two-dimensional and the axisymmetric cases.

Near the stagnation point of a blunt body, the first term in the expansion of the external velocity is $u_e = (du_e/dx)_{x=0}x$. From the transformation equation (2.7) we see that near the stagnation point for two-dimensional flows $\tau \propto x^2$ and $u_e \propto \tau^{1/2}$. For axisymmetric flows, $\tau \propto x^4$ and $u_e \propto \tau^{1/4}$. We than make the second transformation as follows.

1. Two-Dimensional Flows

For two-dimensional flows the transformation is $\tau = \xi^2$ and

$$\frac{\partial}{\partial \tau} = \frac{d\xi}{d\tau}\frac{\partial}{\partial \xi} = \frac{1}{2\xi}\frac{\partial}{\partial \xi}$$

Then the momentum (2.26) and energy (2.31) equations assume the following forms:

Momentum equation:

$$\frac{1}{g_w}\left[\left(\frac{\partial g}{\partial h}\right) h_\eta f_{\eta\eta} + g f_{\eta\eta\eta}\right] + f f_{\eta\eta} + \left(\frac{d\ ln\ u_e}{d\ ln\ \xi}\right)\left(\frac{\delta}{\delta_e} - f_\eta^2\right) = \xi(f_\eta f_{\xi\eta} - f_\xi f_{\eta\eta}) \tag{2.33}$$

Energy equation:

$$g_w\left[\left(\frac{\partial m}{\partial h}\right) h_\eta^2 + m h_{\eta\eta}\right] + \sigma_w m_w g_w f h_\eta$$

$$+ 2\sigma_w m_w \left(\frac{u_e^2}{2H_{st}}\right)\left[\left(\frac{\partial g}{\partial h}\right) h_\eta f_\eta f_{\eta\eta} + g(f_{\eta\eta}^2 + f_\eta f_{\eta\eta\eta}) + g_w f f_\eta f_{\eta\eta}\right]$$

$$- \xi\sigma_w m_w g_w\left[f_\eta h_\xi - h_\eta f_\xi + 2\left(\frac{u_e^2}{2H_{st}}\right)(f_\eta^2 f_{\xi\eta} - f_\xi f_\eta f_{\eta\eta}) + \frac{d}{d\xi}\left(\frac{u_e^2}{2H_{st}}\right) f_\eta^3\right] = 0 \tag{2.34}$$

The transformation from the original equations, (2.1) to (2.4), to (2.33) and (2.34) could have been made in one step by taking:

$$\xi = (\text{sgn } x)\left(\int_o^x \rho_w \mu_w u_e\, dx_1\right)^{1/2} \tag{2.35}$$

$$\eta = \frac{u_e}{\xi\sqrt{2}}\int_o^y \rho\, dy_1 \tag{2.36}$$

2. Axisymmetric Flows

For axially symmetric flows the transformation used in equations (2.26) and (2.31) is $\tau = \xi^4$ and

$$\frac{\partial}{\partial \tau} = \frac{d\xi}{d\tau}\frac{\partial}{\partial \xi} = \frac{1}{4\xi^3}\frac{\partial}{\partial \xi}$$

Then equations (2.26) and (2.31) assume the following forms:
Momentum equation:

$$\frac{1}{g_w}\left[\left(\frac{\partial g}{\partial h}\right) h_\eta f_{\eta\eta} + g f_{\eta\eta\eta}\right] + f f_{\eta\eta} + \frac{1}{2}\left(\frac{d\ \ln u_e}{d\ \ln \xi}\right)\left(\frac{\delta}{\delta_e} - f_\eta^2\right)$$

$$= \frac{1}{2}\xi(f_\eta f_{\xi\eta} - f_\xi f_{\eta\eta}) \qquad (2.37)$$

Energy equation:

$$g_w\left[\left(\frac{\partial m}{\partial h}\right) h_\eta^2 + m h_{\eta\eta}\right] + \sigma_w m_w g_w f h_\eta$$

$$+ 2\sigma_w m_w \left(\frac{u_e^2}{2H_{st}}\right)\left[\left(\frac{\partial g}{\partial h}\right) h_\eta f_\eta f_{\eta\eta} + g(f_{\eta\eta}^2 + f_\eta f_{\eta\eta\eta}) + g_w f f_\eta f_{\eta\eta}\right]$$

$$- \frac{1}{2}\xi\sigma_w m_w g_w\left[f_\eta h_\xi - h_\eta f_\xi + 2\left(\frac{u_e^2}{2H_{st}}\right)(f_\eta^2 f_{\xi\eta} - f_\xi f_\eta f_{\eta\eta}) + \frac{d}{d\xi}\left(\frac{u_e^2}{2H_{st}}\right) f_\eta^3\right] = 0 \qquad (2.38)$$

Again, the transformation from the original equations to (2.37) and (2.38) could have been made in one step by taking:

$$\xi = (\operatorname{sgn} x)\left(\int_0^x \rho_w \mu_w u_e r^2\, dx_1\right)^{1/4} \qquad (2.39)$$

$$\eta = \frac{r u_e}{\xi^2\sqrt{2}} \int_0^y \rho\, dy_1 \qquad (2.40)$$

3. Boundary Conditions

The boundary conditions for the pairs of equations (2.33), (2.34) and (2.37), (2.38) at the wall are

$$f(\xi,0) = f_\eta(\xi,0) = 0 \qquad (2.41a)$$

$$h(\xi,0) = hw(\xi) \quad \text{or} \quad h_\eta(\xi,0) = h_{\eta w}(\xi) \qquad (2.41b)$$

and for the transition to the exterior flow:

$$f_\eta(\xi,\infty) = 1 \qquad (2.41c)$$

$$h(\xi,\infty) = h_e(\xi) = 1 - \frac{u_e^2(\xi)}{2H_{st}} \qquad (2.41d)$$

4. Form of Equations

We see that in the variables ξ, η, the two-dimensional and the axisymmetric equations are slightly different and will have different solutions. In the remainder of this work only the axially symmetric case will be considered.

D. REMARK ON THE MANGLER TRANSFORMATION

We have shown that the equations for two-dimensional and axisymmetric flows are identical in the independent variables τ and η. Following the idea of Mangler (ref. 1, p. 190), we can compare body shapes for identical flows (in τ and η) for the two cases. Specifically, we will check the two-dimensional shape that will correspond to an axisymmetric blunt body. To do this we set up two hypothetical flows with the same $u_e(\tau)$, $\rho_w(\tau)$, and $\mu_w(\tau)$.

We let:

x = longitudinal coordinate with axisymmetry

$\bar{x}$ = longitudinal coordinate with two-dimensional flow

Then

$$\tau = \int_0^x \rho_w \mu_w u_e r^2(x_1) dx_1$$

$$\tau = \int_0^{\bar{x}} \rho_w \mu_w u_e \, d\bar{x}_1$$

So x and $\bar{x}$ have a one-to-one correspondence at corresponding points (same τ values).

For flow around an axisymmetric blunt body we have the external velocity distribution:

$$u_e = \alpha_1 x + \alpha_3 x^3 + \alpha_5 x^5 + \ldots$$

or

$$u_e = \beta_1 \tau^{1/4} + \beta_3 \tau^{3/4} + \beta_5 \tau^{5/4} + \ldots$$

For the corresponding two-dimensional body

$$\frac{d\tau}{d\bar{x}} = \rho_w(\tau)\mu_w(\tau)u_e(\tau)$$

and

$$\bar{x} = \int_0^{\tau} \frac{d\tau_1}{\rho_w \mu_w u_e}$$

which yields

$$\bar{x} \approx c_1 \tau^{3/4} + \ldots$$

and

$$u_e \approx k_1 \bar{x}^{1/3} + \ldots$$

This u_e distribution corresponds to a body with a wedge-shaped vertex, not a blunt body. In most cases this wedge shape would have to be found numerically and it would be tedious. For correspondence in supersonic flows the shock wave should be detached from the wedge. (This will generally be the case; for a 90° wedge the shock wave will be detached for all but the highest free-stream Mach number flows).

Except in the immediate vicinity of the stagnation point, it seems that the Mangler transformation is not of great practical use in making a two-dimensional comparison with an axisymmetric blunt body. In any case, the Mangler transformation does not give a correspondence between a blunt axisymmetric body and a blunt two-dimensional body. This result is similar to that found in incompressible flow, where, for example, the Homann stagnation-point problem corresponds in two dimensions to the 90° wedge (ref. 1, p. 190).

CHAPTER 3

DETERMINATION OF THE STAGNATION-POINT EQUATIONS

The momentum and energy equations (2.37) and (2.38) can be expanded in powers of the transformed longitudinal variable, ξ, as illustrated in appendix A. In this chapter we will obtain the stagnation-point equations which are of zero order in powers of ξ.

A. REPRESENTATION OF THE GAS PROPERTIES

In equations (2.21), (2.24), and (2.28) were introduced dimensionless representations of the gas properties, g, δ, m. The functional forms of the gas properties will affect the solutions of the boundary-layer equations. The gas properties will be represented as polynomials in powers of (h - 1). The coefficients in the polynomials will become parameters to which we can assign values.

$$g(h;\, H_{st},P) = 1 + \sum_{j=1}^{Ng} \frac{g_j}{j!} (h - 1)^j \tag{3.1a}$$

$$m(h;\, H_{st},P) = 1 + \sum_{j=1}^{Nm} \frac{m_j}{j!} (h - 1)^j \tag{3.1b}$$

$$\delta(h;\, H_{st},P) = 1 + \sum_{j=1}^{N\delta} \frac{\delta_j}{j!} (h - 1)^j \tag{3.1c}$$

The quantities Ng, Nm, Nδ denote the highest powers retained in the polynomial expressions. For real gases, the coefficients g_j, m_j, δ_j will depend on the value of H_{st} and they will vary slightly with the pressure, P. For perfect gases, there is no dependence on P, and δ does not depend on H_{st} (see appendix A, sect. C).

B. EXPANSION OF THE QUANTITIES IN THE MOMENTUM AND ENERGY EQUATIONS IN POWERS OF THE LONGITUDINAL VARIABLE

To begin, we expand the functions $f(\xi,\eta)$ and $h(\xi,\eta)$ as follows:

$$f(\xi,\eta) = \sum_{i=0,2,4,\ldots}^{\infty} \frac{f_i(\eta)\xi^i}{i!} \tag{3.2}$$

$$h(\xi,\eta) = \sum_{i=0,2,4,\ldots}^{\infty} \frac{h_i(\eta)\xi^i}{i!} \tag{3.3}$$

These expansions are even in powers of ξ because of the symmetry of the flow. This is obvious in the case of h. With f we recall that $f_\eta = u/u_e$. The velocity, u, must be odd or even according as u_e is odd or even, so their quotient is even. Thus, f_η and f must be even in ξ. (It will be seen below that u_e is odd in ξ.) Using the notation $f_i^{(k)} = [d^k f_i(\eta)]/d\eta^k$, we have

$$\frac{\partial^k f}{\partial \eta^k} = \sum_{i=0,2,4,\ldots}^{\infty} \frac{f_i^{(k)}}{i!}\,\xi^i$$

with corresponding expressions for h.

It is also necessary to expand the external velocity $u_e(\xi)$ in powers of ξ.

$$\frac{u_e}{\sqrt{2H_{st}}} = \sum_{i=1,3,5,\ldots}^{\infty} \frac{a_i}{i!}\,\xi^i \qquad (a_1 \neq 0) \tag{3.4}$$

The a_i will be the parameters that describe the distribution of the external velocity from the stagnation point of a blunt body. From considerations of symmetry of the flow and knowing that u_e is zero at the stagnation point we see that the expansion for u_e must be odd in ξ.

In the momentum equation (2.37), the term $\dfrac{d\ \ln u_e}{d\ \ln \xi}$ occurs; it is expanded to

$$\frac{d\ \ln u_e}{d\ \ln \xi} = \frac{\xi}{u_e}\frac{du_e}{d\xi} = \frac{\xi}{\left(\frac{u_e}{\sqrt{2H_{st}}}\right)}\frac{d}{d\xi}\left(\frac{u_e}{\sqrt{2H_{st}}}\right) = \frac{\sum_{i=1,3,5,\ldots}^{\infty} \frac{a_i}{(i-1)!}\xi^i}{\sum_{i=1,3,5,\ldots}^{\infty} \frac{a_i}{i!}\xi^i} \tag{3.5}$$

In the energy equation (2.38), the term $u_e^2/2H_{st}$ can be expanded as follows

$$\frac{u_e^2}{2H_{st}} = \left(\frac{u_e}{\sqrt{2H_{st}}}\right)^2 = \sum_{n=2,4,6,\ldots}^{\infty} \frac{b_n}{n!}\xi^n \tag{3.6}$$

where (see eq. (3.4))

$$b_n = \sum_{r=1,3,5,\ldots}^{n-1} \binom{n}{r} a_r a_{n-r} \tag{3.7}$$

and $\binom{n}{r}$ is the binomial coefficient.

The wall Prandtl number, σ_w, can be expanded in powers of ξ as:

$$\sigma_w = \sum_{i=0,2,4,\ldots}^{\infty} \frac{\sigma_{wi}}{i!}\xi^i \tag{3.8}$$

Now all the terms needed to expand (2.37) and (2.38) are available.

We next examine the boundary conditions (2.41) in view of the expansions assumed ((3.2) and (3.3)). For the wall-enthalpy boundary condition the expansion is

$$h_w(\xi) = \sum_{i=0,2,4,\ldots}^{\infty} \frac{h_{wi}}{i!}\xi^i \tag{3.9}$$

where the h_{wi} are constant parameters. For the free-stream enthalpy boundary condition we have the expansion

$$h_e(\xi) = 1 - \frac{u_e^2}{2H_{st}} = 1 - \sum_{n=2,4,6,\ldots}^{\infty} \frac{b_n}{n!}\xi^n \tag{3.10}$$

Referring to equations (2.41) and using (3.9), (3.10), the boundary conditions for the f and h expansions can be stated as follows:
At the wall,

$$f_i(0) = f_i'(0) = 0 \qquad i: \ 0, 2, 4, \ldots \tag{3.11a}$$

$$h_i(0) = hwi \qquad i: \ 0, 2, 4, \ldots \tag{3.11b}$$

and at the transition to the exterior flow,

$$f_o'(\infty) = 1 \quad \text{and} \quad f_i'(\infty) = 0 \qquad i: \ 2, 4, 6, \ldots \tag{3.11c}$$

$$h_o(\infty) = 1 \quad \text{and} \quad h_i(\infty) = -b_i \qquad i: \ 2, 4, 6, \ldots \tag{3.11d}$$

In the case of a prescribed heat flux, the boundary condition $h_i'(0)$ will be given instead of (3.11b)

C. THE STAGNATION-POINT EQUATIONS

These equations are obtained by considering terms of zero order, ξ^o. To write these equations we set $\xi = 0$ in (2.37) and (2.38). We will use the notation, $g_{wo} = g(hwo)$ and $m_{wo} = m(hwo)$. From (3.5)

$$\left(\frac{d \ln u_e}{d \ln \xi}\right)_{\xi=o} = 1$$

From equations (2.24b), (3.1c), and (3.10) we have:

$$\delta_{e(\xi=o)} = \delta[1; H_{st}, P(0)] = 1$$

Then the zero-order equations are:
Momentum equation:

$$\frac{1}{g_{wo}}\left\{\left(\frac{\partial g}{\partial h}\right)_{h_o} h_o' f_o'' + [g(h_o)] f_o'''\right\} + f_o f_o'' + \frac{1}{2}[\delta(h_o) - (f_o')^2] = 0 \tag{3.12}$$

Energy equation:

$$[m(h_o)]h_o'' + \left(\frac{\partial m}{\partial h}\right)_{h_o} (h_o')^2 + \sigma_{wo} m_{wo} f_o h_o' = 0 \tag{3.13}$$

Equations (3.12) and (3.13) are a pair of coupled nonlinear ordinary differential equations in f_o and h_o. The boundary conditions are

given by (3.11) for $i = 0$. These are the equations to which we will seek general type solutions. The solutions for f_o and h_o can be considered as valid representations of f and h in the vicinity of the stagnation point, the first neglected terms being of order ξ^2.

CHAPTER 4

THE METHOD OF SOLUTION OF THE STAGNATION-POINT EQUATIONS IN TERMS OF THE CONTROLLING PARAMETERS

In this chapter we will derive the method to be used in obtaining universal type solutions to the stagnation-point equations. The numerical results will be presented in chapter 5.

A. THE STAGNATION-POINT EQUATIONS

At and very near the stagnation point the values of f and h become f_o and h_o, respectively (eqs. (3.2) and (3.3)). These are obtained as solutions to the zero-order equations (3.12), (3.13).

At this point, in order to avoid excessive subscripting we will make the following change of nomenclature:

$$f_o = F \tag{4.1a}$$

$$h_o = G \tag{4.1b}$$

We will introduce polynomials (3.1) into (3.12), (3.13), and we will assume one new relation:

$$\sigma_{wo} = \sigma_o + \sigma_1(hwo - 1) \tag{4.2}$$

Equation (4.2) is an artificial relationship between constants. It will be seen subsequently that the use of (4.2) permits the perturbation of σ_{wo} to be combined with that of hwo. The quantity, σ_o, is the reference value around which σ_{wo} is perturbed, and it is the wall Prandtl number value used in obtaining machine solutions for universal functions, while σ_1 is defined by equation (4.2). It will be seen that we will always work with the product $\sigma_1(hwo - 1)$ which remains finite.

We now have for the zero-order equations

Energy equation:

$$\left[1 + \sum_{j=1}^{Nm} \frac{m_j}{j!} (G - 1)^j\right] G'' + \left[m_1 + \sum_{j=2}^{Nm} \frac{m_j}{(j - 1)!} (G - 1)^{j-1}\right] (G')^2$$

$$+ [\sigma_o + \sigma_1(hwo - 1)] \left[1 + \sum_{j=1}^{Nm} \frac{m_j}{j!} (hwo - 1)^j\right] FG' = 0 \qquad (4.3)$$

Momentum equation:

$$\left[g_1 + \sum_{j=2}^{Ng} \frac{g_j}{(j - 1)!} (G - 1)^{j-1}\right] G'F'' + \left[1 + \sum_{j=1}^{Ng} \frac{g_j}{j!} (G - 1)^j\right] F'''$$

$$+ \left[1 + \sum_{j=1}^{Ng} \frac{g_j}{j!} (hwo - 1)^j\right] \left\{FF'' - \frac{1}{2} (F')^2 + \frac{1}{2} \left[1 + \sum_{j=1}^{N\delta} \frac{\delta_j}{j!} (G - 1)^j\right]\right\} = 0 \qquad (4.4)$$

In the new notation the boundary conditions (3.11) are as follows:

At the wall:

$$F(0) = F'(0) = 0 \qquad (4.5a)$$

$$G(0) = hwo \qquad (4.5b)$$

and at the transition to the exterior flow:

$$F'(\infty) = G(\infty) = 1 \qquad (4.5c)$$

B. METHOD OF SOLUTION

We will attempt to set up universal type solutions of (4.3) and (4.4) in terms of the controlling parameters; in particular we will expand G and F into Taylor's series in terms of (hwo - 1). It will be seen below that the effects of all the other parameters will be separated out by the perturbation of the one parameter, hwo.

In Taylor series form we have

$$G(\eta) = G_o(\eta) + \sum_{i=1}^{\infty} \frac{G_i(\eta)}{i!} (hwo - 1)^i \qquad (4.6)$$

$$F(\eta) = F_o(\eta) + \sum_{i=1}^{\infty} \frac{F_i(\eta)}{i!} (hwo - 1)^i \qquad (4.7)$$

where

$$G_i(\eta) = \left[\frac{\partial^{(i)} G(\eta)}{\partial hwo^{(i)}} \right]_{(hwo=1)}$$

with a corresponding expression for $F_i(\eta)$. From equations (4.5) we can write the boundary conditions for the terms of (4.6) and (4.7) as follows:

At the wall:

$$F_i(0) = F_i'(0) = 0 \qquad i: \ 0, 1, 2, 3, \ldots \qquad (4.8a)$$

$$G_o(0) = G_1(0) = 1 \qquad (4.8b)$$

$$G_i(0) = 0 \qquad i: \ 2, 3, 4, \ldots \qquad (4.8c)$$

and at the transition to the exterior flow:

$$F_o'(\infty) = 1 \quad F_i'(\infty) = 0 \qquad i: \ 1, 2, 3, \ldots \qquad (4.8d)$$

$$G_o(\infty) = 1 \quad G_i(\infty) = 0 \qquad i: \ 1, 2, 3, \ldots \qquad (4.8e)$$

The expansion of G and F in powers of (hwo - 1), around hwo = 1, is a regular or nonsingular perturbation. It is a perturbation around a given boundary condition. The forms of equations (4.3) and (4.4) are preserved and no boundary conditions are lost.

The range of convergence of expansions (4.6) and (4.7) is not known a priori. This question will be discussed in chapter 5 in connection with the numerical solutions obtained.

C. ZERO-ORDER, $(hwo - 1)^o$, EQUATIONS

To obtain these equations, we substitute expansions (4.6), (4.7) into equations (4.3), (4.4) and set hwo = 1. We obtain:

Energy equation:

$$\left[1 + \sum_{j=1}^{Nm} \frac{m_j}{j!} (G_o - 1)^j\right] G_o'' + \left[m_1 + \sum_{j=2}^{Nm} \frac{m_j}{(j-1)!} (G_o - 1)^{j-1}\right] (G_o')^2 + \sigma_o F_o G_o' = 0 \qquad (4.9)$$

Momentum equation:

$$\left[g_1 + \sum_{j=2}^{Ng} \frac{g_j}{(j-1)!} (G_o - 1)^{j-1}\right] G_o' F_o'' + \left[1 + \sum_{j=1}^{Ng} \frac{g_j}{j!} (G_o - 1)^j\right] F_o''' + F_o F_o'' - \frac{1}{2} (F_o')^2 + \frac{1}{2}\left[1 + \sum_{j=1}^{N\delta} \frac{\delta_j}{j!} (G_o - 1)^j\right] = 0 \qquad (4.10)$$

The boundary conditions are given in equations (4.8) with i = 0.

It can be seen by inspection that the energy equation (4.9) has the solution

$$G_o \equiv 1 \qquad (4.11)$$

We insert (4.11) into (4.10) and obtain:

$$F_o''' + F_o F_o'' + \frac{1}{2}\left[1 - (F_o')^2\right] = 0 \qquad (4.12)$$

Equation (4.12) with boundary conditions given by (4.8a) and the left side of (4.8d) is the Homann equation for the axisymmetric stagnation-point problem for incompressible flow. The numerical solution of the Homann equation is known (ref. 1). This is the only nonlinear equation in the system we are developing. All higher-order equations will be seen to be linear.

D. HIGHER-ORDER, $(hwo - 1)^i$, EQUATIONS

The higher-order equations can be determined by substituting expansions (4.6), (4.7) into equations (4.3) and (4.4) and then collecting powers $(hwo - 1)^i$. An equivalent procedure is to take partial derivatives with respect to hwo, to the various orders wanted, of equations (4.3), (4.4) and then to set hwo = 1. The equations all have the following form:

Energy equations:

$$G_i'' + (\sigma_o F_o)G_i' = (\text{right side}) \qquad i: \ 1, 2, 3, \ldots \tag{4.13}$$

Momentum equations:

$$F_i''' + (F_o)F_i'' - (F_o')F_i' + (F_o'')F_i = (\text{right side})$$

$$i: \ 1, 2, 3, \ldots \tag{4.14}$$

The invariance of the left-hand sides of the equations eases somewhat the task of machine programming the solutions of the equations.

Another point of interest that we observe from (4.8) is that, above the first order, all the pertinent boundary conditions are homogeneous. This means that we can always satisfy the boundary conditions of the stagnation-point problem when we carry the expansions to, or beyond the first order, and then truncate the series.

Equations (4.13) and (4.14) are written out with their right-hand sides in appendix B. The solution for each order depends on the solutions of the lower orders. For a given order, the momentum equation is coupled to the energy equation, but not vice versa. As seen in appendix B, the effects of the parameters appear on the right-hand sides of the equations, so the solutions will logically assume a split form.

In obtaining numerical solutions we will use third-degree polynomial representations for the gas properties, m, g, δ. As seen in appendix B, the parameters, m_i, g_i, δ_i, appear progressively with the order of the equations. Through the third order the equations have been set up in complete form, but the numerical results (ch. 5)

will show that some terms can be dropped. Above the third order, only significant terms have been retained in the equations (determined by comparison with exact solutions).

E. FORM OF SERIES FOR G AND F

Solutions of equations (4.12), (4.13), and (4.14) will yield universal functions that can be put into expansions (4.6), (4.7) to yield the following form

$$G = 1 + {}_1G_1 \frac{(hwo - 1)}{1!} + [(m_1){}_1G_2 + (g_1){}_2G_2 + (\delta_1){}_3G_2 + (2\sigma_1){}_2G_1] \frac{(hwo - 1)^2}{2!} + (\text{higher order terms}) \tag{4.15}$$

$$F = F_o + [(g_1){}_1F_1 + (\delta_1){}_2F_1] \frac{(hwo - 1)}{1!} + [\ldots + (g_2){}_4F_2 + (\delta_2){}_5F_2 + (g_1\sigma_1){}_aF_2 + (\delta_1\sigma_1){}_bF_2 + \ldots] \frac{(hwo - 1)^2}{2!} + (\text{higher-order terms}) \tag{4.16}$$

The nomenclature ${}_iG_j$, ${}_iF_j$ represents the ith split function of the jth order. It is multiplied by the parameter to which it corresponds, as indicated on the right-hand side of its equation in appendix B. A letter subscript (e.g., ${}_aF_2$) represents a function that will either be dropped or approximated. An exception to this nomenclature system is with the term ${}_2G_1$ which is a second-order term (see appendix B, sect. B). The nomenclature of this term will appear consistent in equation (4.17) below. The term ${}_2G_1$ has the boundary condition of the second-order terms, ${}_2G_1(0) = 0$.

Using equation (4.2) we substitute $(\sigma_{wo} - \sigma_o)$ for $\sigma_1(hwo - 1)$ in the series (4.15), (4.16) and we have the more desirable form

$$G = 1 + \left[{}_1G_1 + (\sigma_{wo} - \sigma_o)\,{}_2G_1 + \frac{1}{6}(\sigma_{wo} - \sigma_o)^2\,{}_aG_3 + \text{higher orders in } (\sigma_{wo} - \sigma_o)^n \right] \frac{(hwo - 1)}{1!} + \left[(m_1)\,{}_1G_2 + (g_1)\,{}_2G_2 + \ldots \right] \frac{(hwo - 1)^2}{2!} + (\text{higher-order terms}) \tag{4.17}$$

$$F = F_o + \left\{ \left[{}_1F_1 + \frac{1}{2}(\sigma_{wo} - \sigma_o)\,{}_aF_2 + \frac{1}{6}(\sigma_{wo} - \sigma_o)^2\,{}_aF_3 \right] (g_1) + \left[{}_2F_1 + \frac{1}{2}(\sigma_{wo} - \sigma_o)\,{}_bF_2 + \frac{1}{6}(\sigma_{wo} - \sigma_o)^2\,{}_bF_3 \right] (\delta_1) + \text{higher orders in } (\sigma_{wo} - \sigma_o)^n \right\} \frac{(hwo - 1)}{1!} + \left[\ldots + (g_2)\,{}_4F_2 + (\delta_2)\,{}_5F_2 + \ldots \right] \frac{(hwo - 1)^2}{2!} + (\text{higher-order terms}) \tag{4.18}$$

In obtaining numerical solutions we will restrict the values that we will allow σ_{wo} to take within rather narrow practical limits ($0.65 \leq \sigma_{wo} \leq 0.80$). Comparison with exact solutions will show that within any given order of $(hwo - 1)^n$, orders of $(\sigma_{wo} - \sigma_o)^j$ higher than the first can be neglected. In fact, for some of the terms, even the first order of $(\sigma_{wo} - \sigma_o)$ produces a negligible effect.

The series indicated above (eqs. (4.17), (4.18)) are written in their final form in chapter 5 (eqs. (5.3), (5.4)). At this point it should be mentioned that we will never use the infinite series. We will truncate the series, so that actually G and F are represented by finite polynomials in powers of $(hwo - 1)^j$.

For the convenience of the reader, in figure 1 are shown the derivation of the ${}_iG_j$ and ${}_iF_j$ terms through the third order, and the significant (retained) terms are shown for the higher orders.

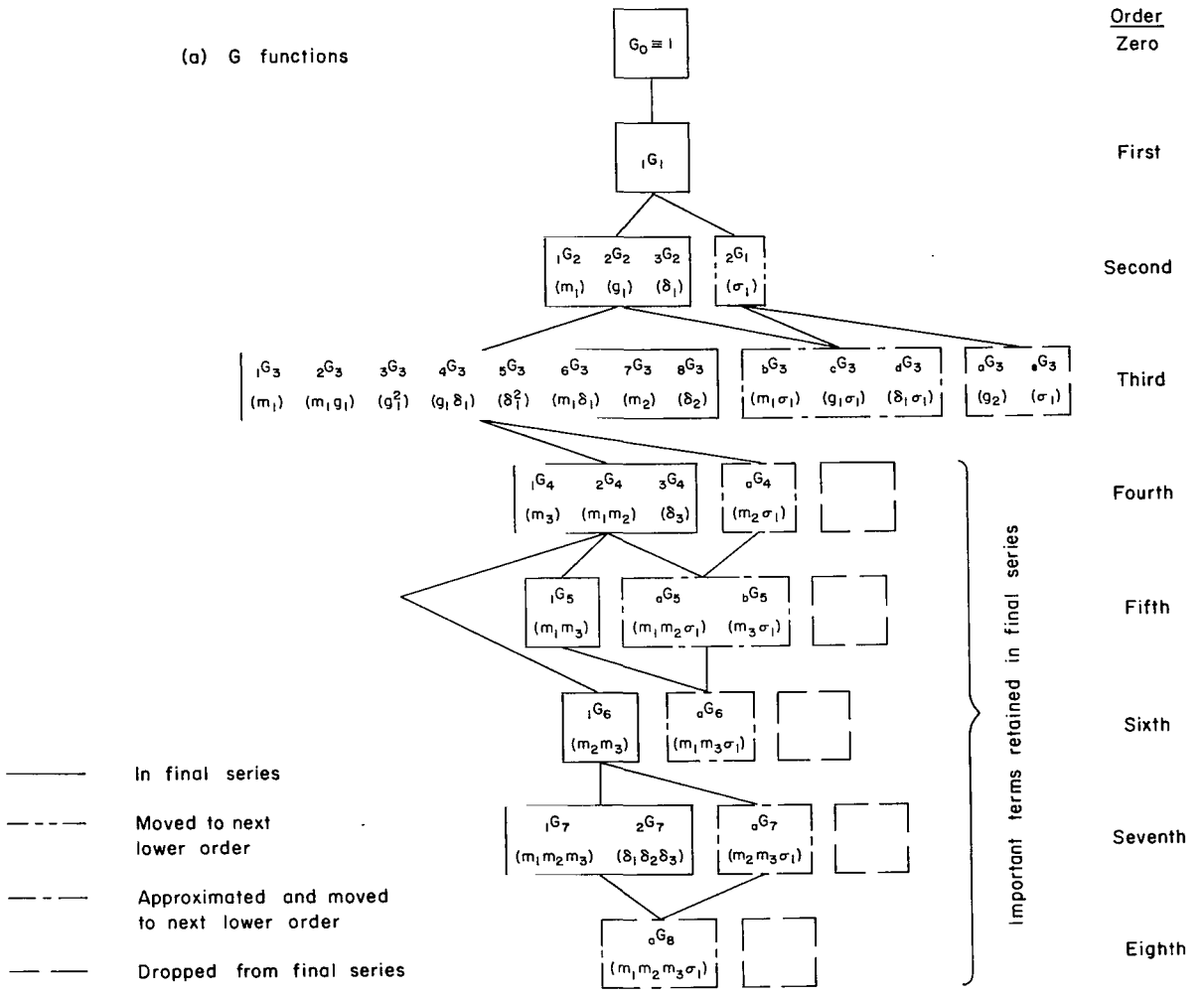


Figure 1.- Derivation of axisymmetric stagnation-point universal functions.

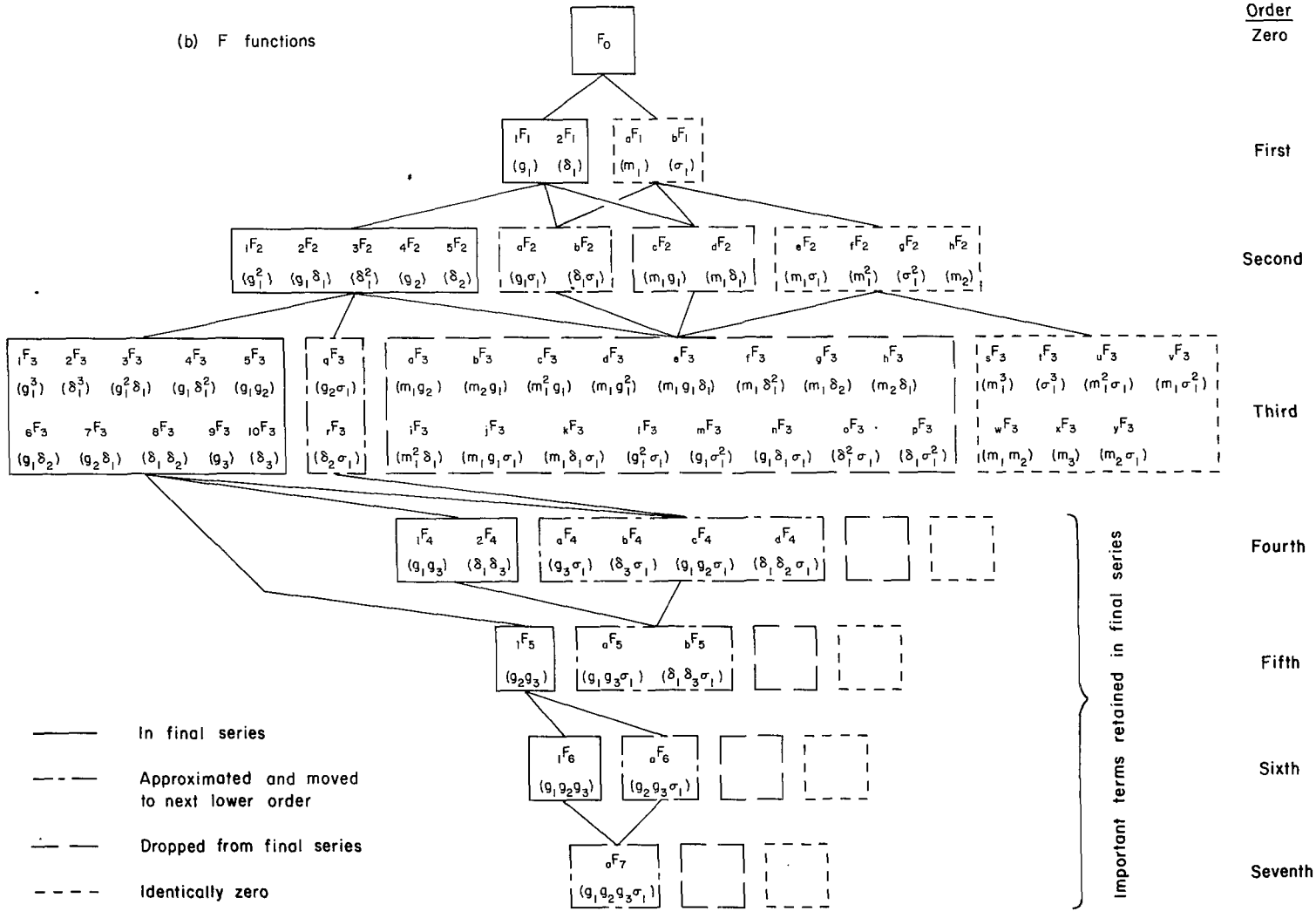


Figure 1.- Concluded.

F. THE METHOD OF SOLUTION APPLIED TO EQUATIONS OF HIGHER ORDER IN POWERS OF ξ

A method of determining general solutions to the stagnation-point (zero order in ξ) equations has been explained in this chapter. This same procedure of obtaining solutions by expanding in powers of $(hwo - 1)^j$ can be applied to the equations of higher order in ξ, the forms of which are given in appendix A. These solutions become increasingly complicated, as they depend on the solutions to the zero-order (and lower ξ^n order) equations. It is probably more practical to obtain solutions downstream of the stagnation point by finite-difference methods. The procedure described here will determine stagnation-point profiles which can be used as starting values for finite-difference methods.

CHAPTER 5

NUMERICAL SOLUTIONS FOR THE STAGNATION-POINT EQUATIONS

The equations shown in chapter 4 (and appendix B) cannot be solved analytically. In this chapter we will present the numerical solutions to these equations.

A. METHOD OF OBTAINING NUMERICAL SOLUTIONS

Equations (4.12), (4.13), (4.14) (see also appendix B) with boundary conditions given by (4.8) were programmed by the author for solution on the IBM 7090 digital computer. The solutions to the equations were split as indicated in appendix B. All are linear equations except the zero-order momentum equation (4.12). The machine computation is of such a nature that functions can be generated that solve the differential equations provided sufficient initial (wall boundary) conditions are known. All of the equations to be solved are accompanied by outer boundary conditions rather than just wall boundary conditions. Once a solution is determined that satisfies the outer boundary conditions, then sufficient initial conditions are known to generate the solution again.

Because of its nonlinearity and because F_o and its derivatives are in the coefficients and the right-hand sides of the linear equations (4.13), (4.14), equation (4.12), the Homann equation, was solved separately from the rest. The procedure in finding the solution was to adjust an unknown initial condition, $F_o''(0)$, until the outer boundary condition (left side of eq. (4.8d)) was satisfied. Then the function could be generated again using the three initial conditions, $F_o(0)$, $F_o'(0)$, $F_o''(0)$. The purpose of this solution was to accurately determine $F_o''(0)$. Then, in solving the linear equations, the computing machine could generate $F_o(\eta)$ and its derivatives as needed for solutions with a variable increment size (see sect. B below). For the machine computation, infinity was taken as sufficiently realized at $\eta = 10$. This seems allowable as $F_o'(\eta)$ approaches unity asymptotically as η becomes large. The selection of $\eta = 10$ is explained in

appendix C wherein an asymptotic solution of this equation is given. The plot of $F_o'(\eta)$ is shown in figure 2(a).

The linear equations were solved for the proper boundary values by linear superposition. The complete equations were solved with the given initial conditions, and one additional (arbitrary) assumed initial condition, ${}_iG_j'(0)$ for the energy equations, and ${}_iF_j''(0)$ for the momentum equations. The homogeneous part of the energy equations was solved with the initial conditions, ${}_iG_j(0) = 0$ and with ${}_iG_j'(0)$ begin given an assumed arbitrary value. The homogeneous part of the momentum equations was solved with ${}_iF_j(0) = {}_iF_j'(0) = 0$ and with ${}_iF_j''(0)$ being given an assumed arbitrary value. The homogeneous solutions could then be multiplied by constants and superposed to the complete solutions to obtain the proper boundary conditions at infinity ($\eta = 10$). These superpositions provided the correct missing initial conditions, ${}_iG_j'(0)$ and ${}_iF_j''(0)$. The correct solutions could then be generated again at will. The superposition method described was programmed into the computing procedure.

Because of their dependence on one another, the equations had to be solved in the order in which they are listed in appendix B. Actually, all the equations were programmed into the machine simultaneously (after having separately determined $F_o''(0)$ as described above). The proper order of solution of the equations was handled by the computing machine by letting the program make a number of "passes" through the machine. The final pass generated the correct solutions to all the equations, with the solutions split into their components. All of the solutions approach their boundary condition at infinity asymptotically, and the boundary conditions at infinity are adequately represented at $\eta = 10$.

In programming the solutions to the equations, the author was able to use an existing subroutine for the integration of a family of second-order differential equations. This is "Share" Subroutine RWDE6F converted to "FORTRAN" language. This subroutine obtains starting values by the Runge-Kutta method, and the main integration is carried out by the Livermore Cowell method, which is a predictor-corrector procedure.

This subroutine was used directly in the energy equations which are of second order. An additional differential equation was paired with each energy equation in order to obtain the integral with respect to η of each enthalpy term, ${}_iG_j$ (used in calculating y and δ^* and explained in chapter 6). An existing subroutine for integrating third-order differential equations was not known to the author, so the momentum equations (which are third order) were split into pairs of second-order equations in order to use Subroutine RWDE6F. The simultaneous solution of a large number of coupled differential equations poses no difficulty for the computing machine.

B. THE NUMERICAL RESULTS

The universal solutions obtained separate out the effects of the parameters of the problem (through third-degree gas-property polynomials). The one parameter that cannot be separated out is σ_o because it appears in a coefficient on the left side of the energy equations (4.13). The quantity, σ_o, can be assigned an arbitrary value as it is the reference value around which we elect to perturb the wall Prandtl number, σ_{wo}. For the numerical solutions, we are using the value, $\sigma_o = 0.75$.

A number of terms in the series expansions (4.17) and (4.18) are affected by the value of the wall Prandtl number, σ_{wo}. We have set narrow limits for the variation of this quantity $(0.65) \leq \sigma_{wo} \leq 0.80)$, and within this limitation we can approximate the universal functions that involve σ_{wo} (or σ_1). Each term involving σ_{wo} can be regarded as a modification or perturbation of another term. For example, from equation (4.18) we have the term:

$$\left[{}_1F_1 + \frac{1}{2}(\sigma_{wo} - \sigma_o){}_aF_2 + \frac{1}{6}(\sigma_{wo} - \sigma_o)^2 {}_aF_3 \right] (g_1)$$

Comparison with exact solutions will show that the term involving $(\sigma_{wo} - \sigma_o)^2$ can be neglected while the term with $(\sigma_{wo} - \sigma_o)$ can be considered as a small modification of the first term, ${}_1F_1$. The functions, ${}_1F_1$ and ${}_aF_2$, have the same boundary conditions so we can set up the approximation

$${}_aF_2 \approx (C){}_1F_1$$

where C is a constant to be determined. Then we can write for the term giving the effect of g_1:

$$[1 + C(\sigma_{wo} - \sigma_o)](g_1)_1F_1$$

This is the form that will be used in the final writing of the series for G and F. This approximation states that the effect of σ_{wo} is to modify the effective value of the parameter involved (g_1 in the case illustrated). The accuracy of this approximation was checked by comparison with exact solutions as described below in this chapter. The constant, C, was adjusted to give accurate values of $G'(0)$ and $F''(0)$. This procedure eliminated the need for solving for the universal functions containing σ_1 in the equations in appendix B. The one exception is ${}_2G_1$ which appears in equation (4.17); ${}_2G_1$ was solved for. This was necessary because ${}_2G_1$ has a different boundary condition than ${}_1G_1$, so these two terms could not be combined. The term, ${}_2G_1$, which is actually a second-order term moved to the first order, has the second-order boundary condition ${}_2G_1(0) = 0$ (see ch. 4, sect. E, and appendix B, sect. B).

The universal functions used in the final formulas for G and F are shown in figure 2. The components of the dimensionless velocity, ${}_iF_j'$, have been plotted rather than ${}_iF_j$ as it is felt that ${}_iF_j'$ is more immediately meaningful. In figure 3 are shown plotted the universal integrals (see ch. 6) of the ${}_iG_j$ functions, as defined by the formula

$$ {}_iI_j = \int_o^\eta {}_iG_j \, d\eta_1 \qquad (5.1)$$

In both figures 2 and 3 it is seen that all of the functions approach their outer boundary conditions asymptotically.

Numerical values of the universal functions used are presented in table I. The tabulations include the ${}_iG_j$ functions, the first and second derivatives, and the integrals, ${}_iI_j$, of the ${}_iG_j$ functions; the ${}_iF_j$ functions with the first, second, and third derivatives are tabulated. The machine computes to eight significant figures, but additions and subtractions were made in solving the differential equations;

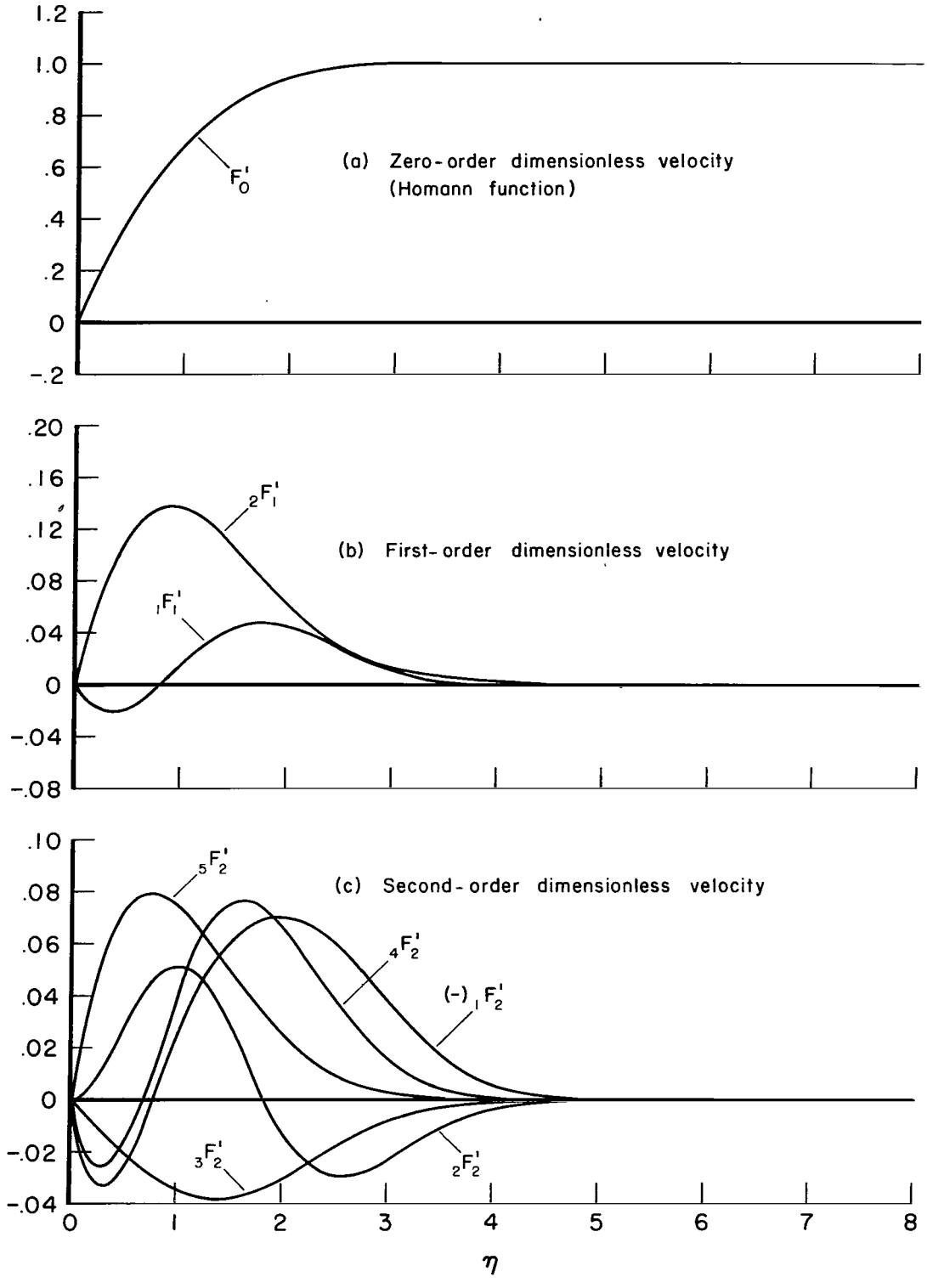


Figure 2.- Axisymmetric stagnation-point universal functions; $\sigma_o = 0.75$.

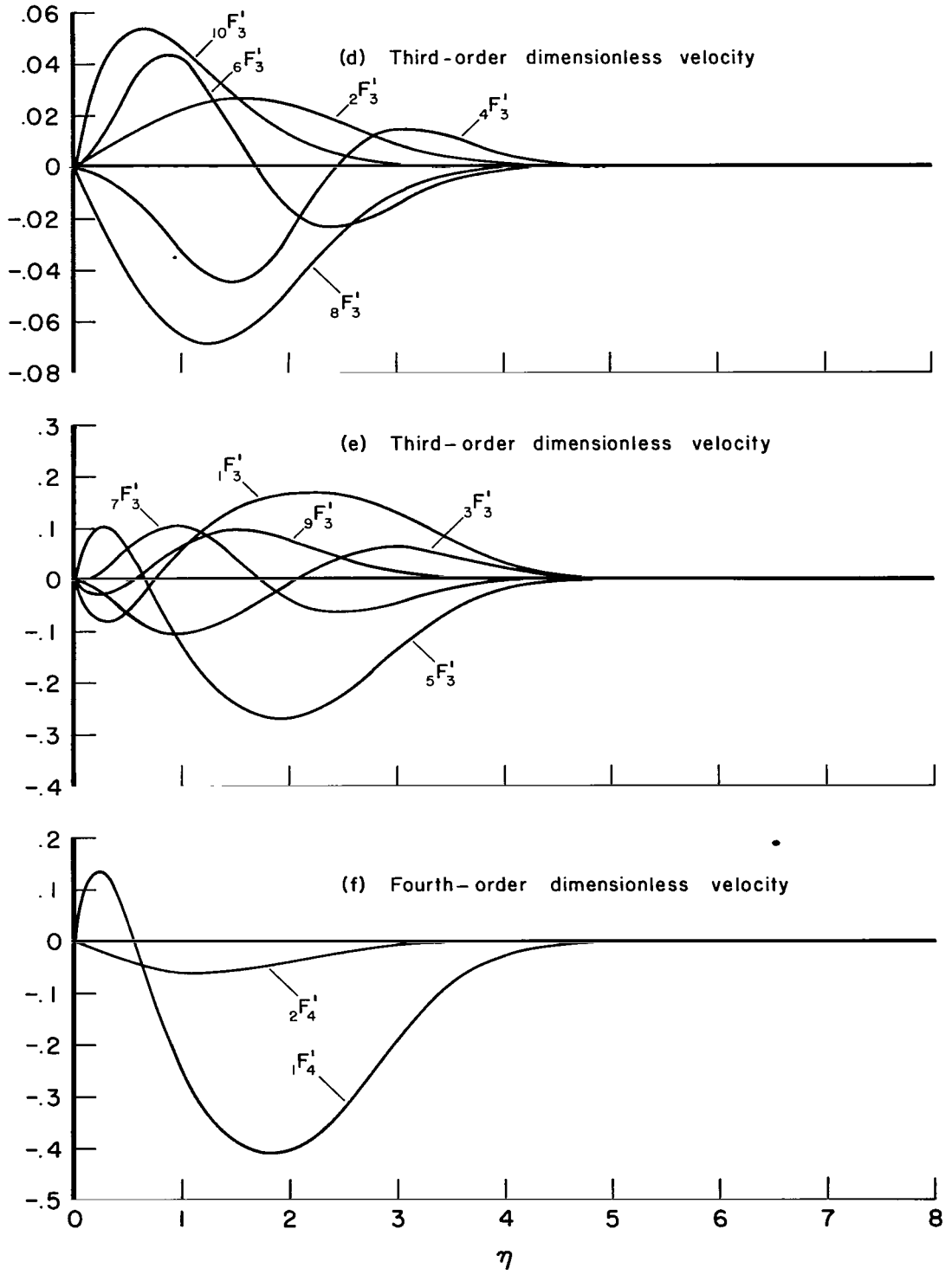


Figure 2.- Continued.

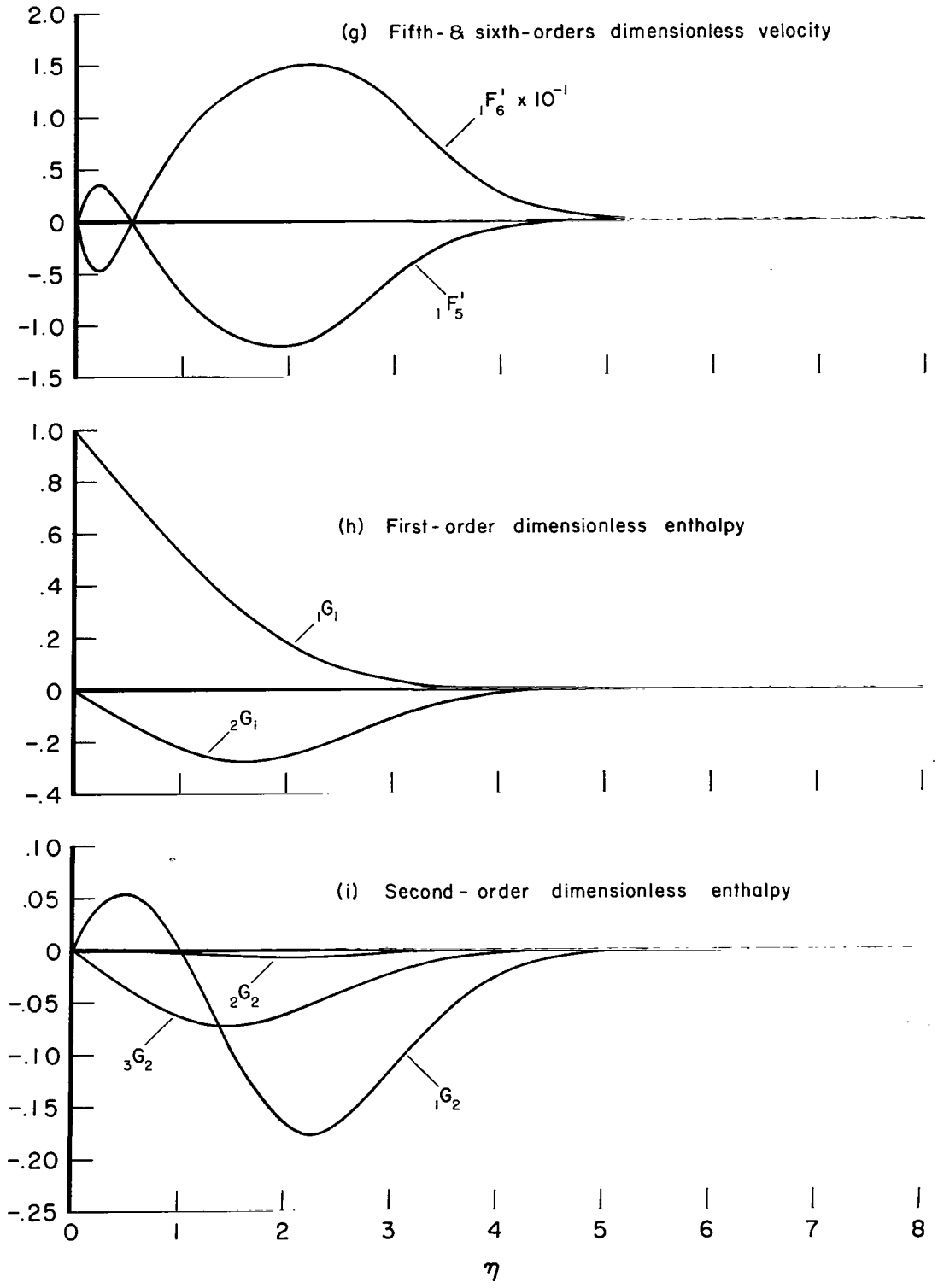


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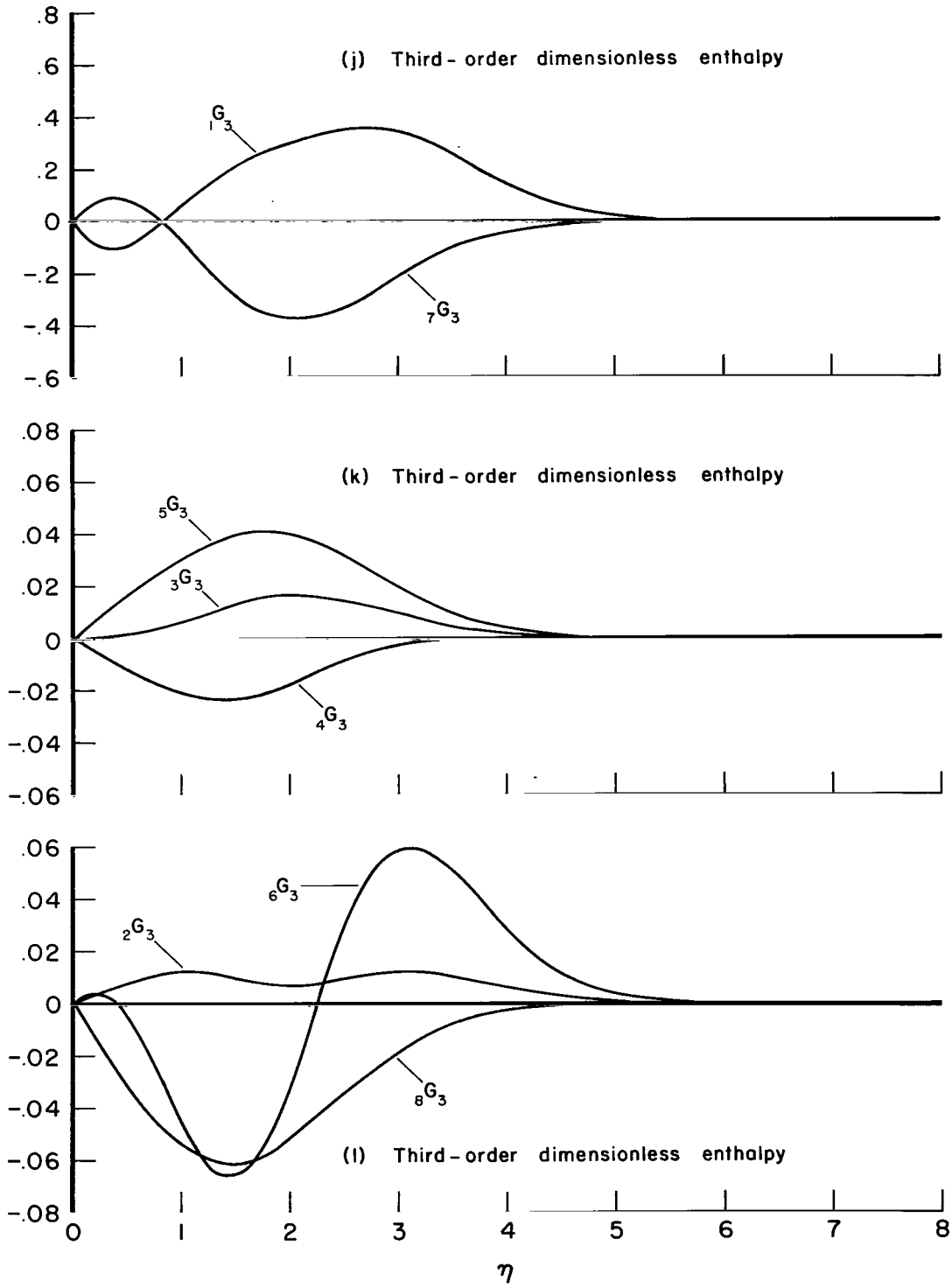


Figure 2.- Continued.

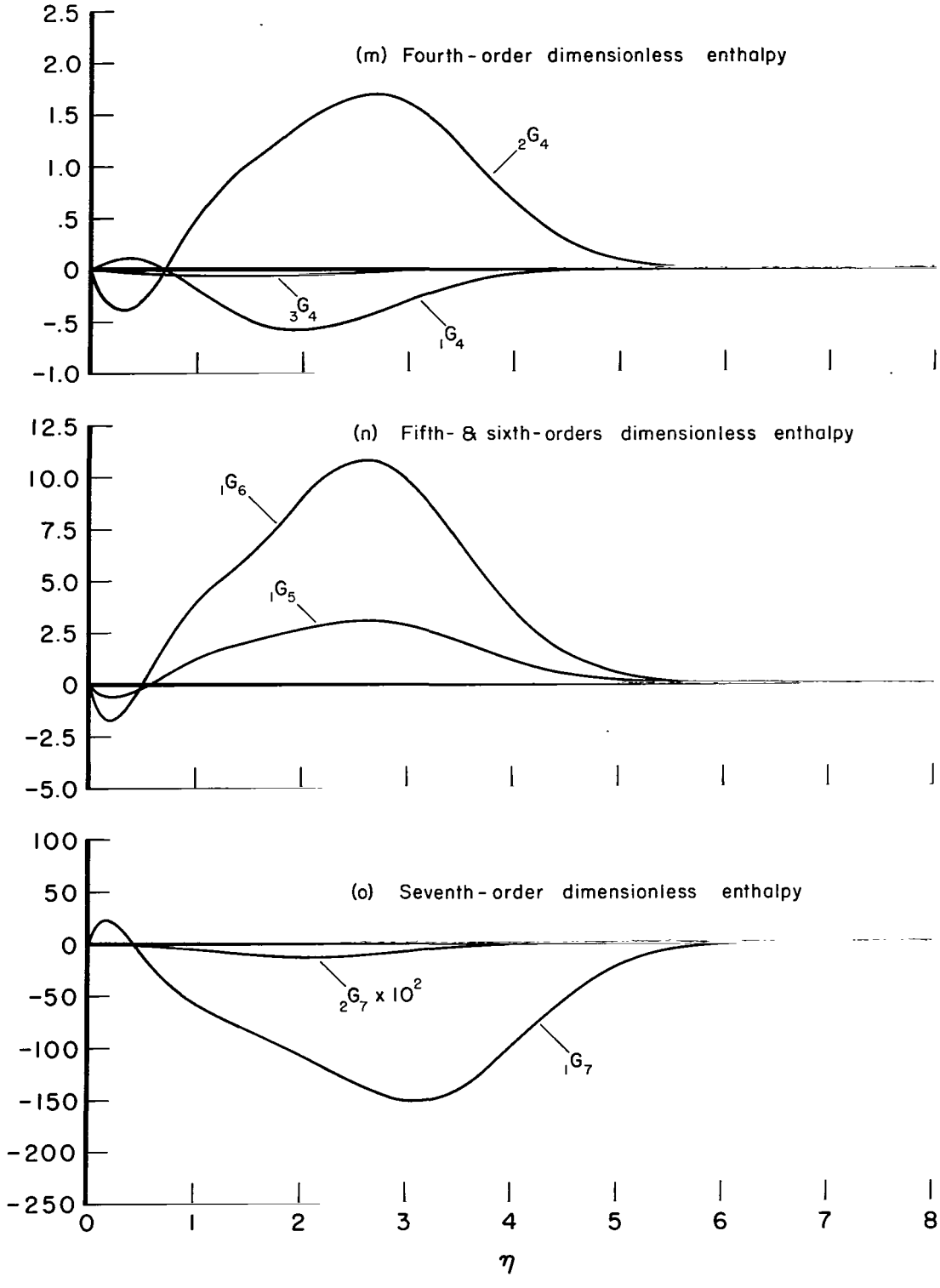


Figure 2.- Concluded.

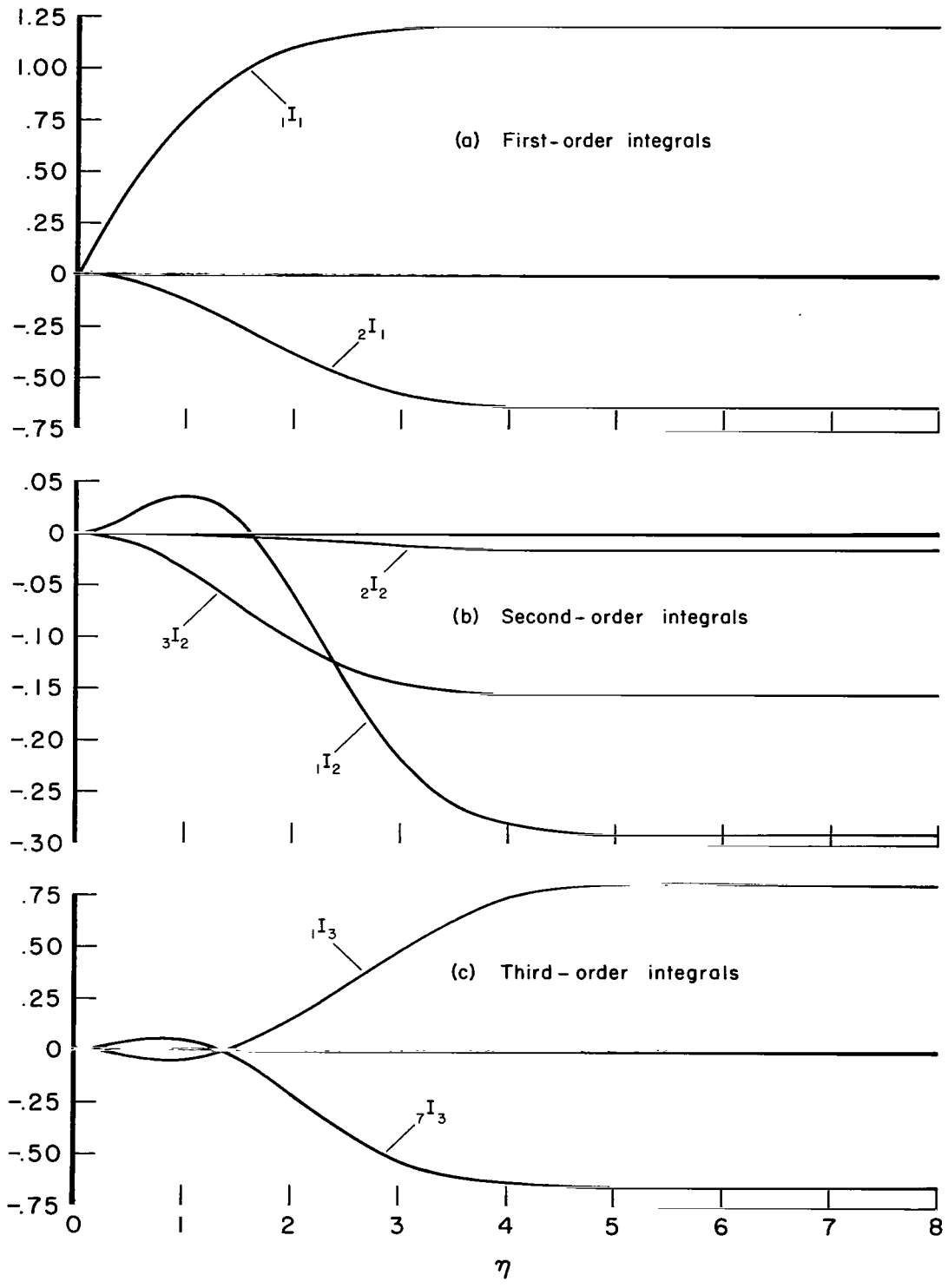


Figure 3.- Axisymmetric stagnation-point universal enthalpy integrals; $\sigma_o = 0.75$.

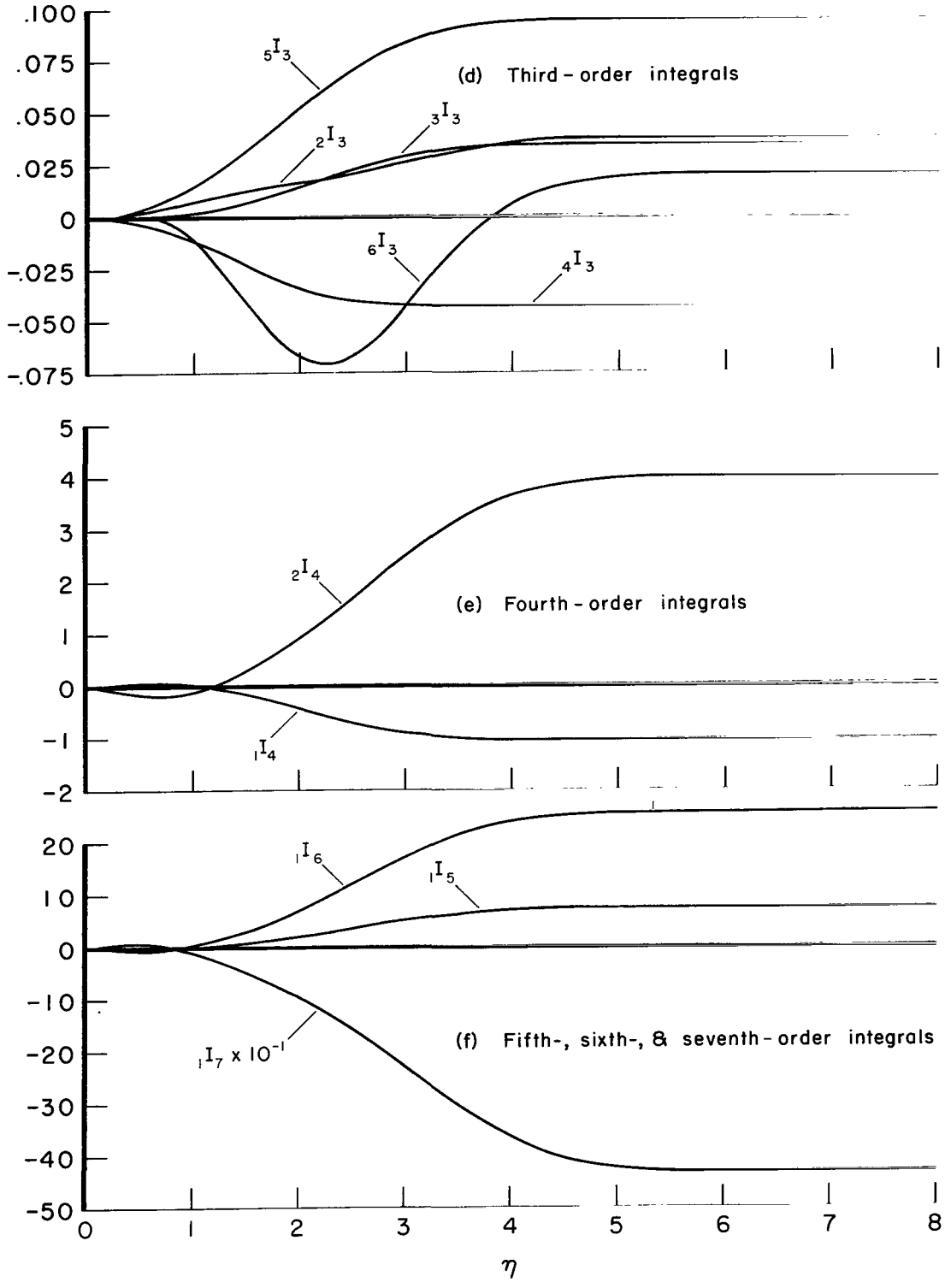


Figure 3.- Concluded.

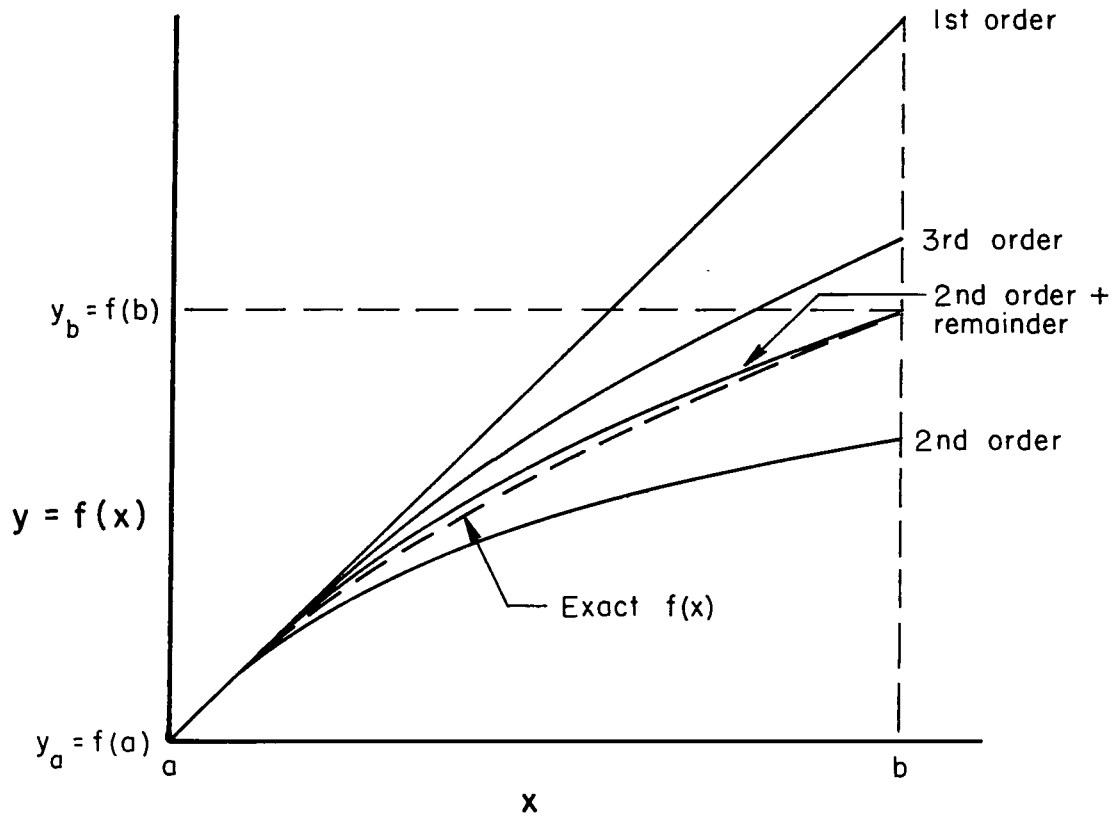
therefore it is believed that the results presented are generally accurate to five decimal places, as presented in table I.

All of the universal functions obtained have been recorded on a permanent binary tape. The differential equations were originally solved with a variable increment size which allows the computing machine to determine optimum increments of the independent variable. In order to avoid tabulating odd values of η, the calculations were changed to fixed increment sizes for recording on the permanent binary tape. The fixed increments were selected to be no larger than the variable increments for given values of η. The smallest increment of η at the wall is 0.0025; moving out from the wall the increment is doubled four times at different η values so that the largest increment is 0.04. Approximately 600 values of η (with corresponding universal functions) are on the permanent binary tape of which a relatively small portion is presented in table I.

C. ADJUSTMENT OF HIGHEST-ORDER TERMS

The series forms for G and F, indicated in equations (4.17) and (4.18) can be thought of as truncations of infinite series. The radii of the circles of convergence in the complex hwo plane of the various terms of the two series are difficult to ascertain, but the principal interest is that the two truncated series accurately represent functions which solve equations (4.3) and (4.4) with boundary conditions (4.5). We will adjust the highest-order terms that we retain (for each parameter or combination of parameters), and the resultant two series can be thought of as Taylor series with remainders.

From the theory of Taylor series with a remainder (ref. 8, p. 98, ref. 9, p. 14), we know that the remainder can be adjusted so the series passes through a given point. As illustrated in the sketch, we can expand the function $y = f(x)$ into a Taylor series around the point, $x = a$. We assume for the present that the series is convergent. We imagine the series carried through the second order and having a remainder term. This remainder term is actually an adjusted third-order term. This adjustment can be made so that the series passes through the exact point, y_b. (The series also passes through y_a since it is expanded



around this point.) For the range of x, $a < x < b$, the fit between the second-order series with remainder and the function, $f(x)$, will, in general, not be exact, but it can be very good depending on how well the function, $f(x)$, can be fitted by a polynomial of third degree. The illustration given is with the assumption that the Taylor series converges when $a \leq x \leq b$. Often, the same considerations will apply when the point, $x = b$, is outside the circle of convergence of the Taylor series, or when the series is an asymptotic expansion. The principal criterion is, again, how well the function, $f(x)$, can be fitted by a third-degree polynomial when the coefficient of the third-degree term is at one's disposal to adjust.

The method explained above was used in adjusting the highest-order terms in series of the type (4.17), (4.18). Practical extremes of (hwo - 1) and the various parameters were selected, and the highest-order terms were adjusted so that the series solutions matched the

exact solutions of equations (4.3) and (4.4) for these extreme conditions. After the adjustments were made to the highest-order terms, moderate combinations of (hwo - 1) and the parameters were then selected and the matchings of the adjusted series solutions with the exact solutions were checked. This will be explained below in more detail.

It is probably desirable to list the steps in a rigorous method of adjustment of the highest-order terms (although this procedure was not actually used). In the sketch above it should be recognized that the variable x corresponds to the product of (hwo - 1) and some combination of parameters. The sketch can be thought of as illustrating the adjustment for one particular value of the variable, η. Of course, this adjustment must be made for all values of η, or (as is probably obvious) the adjusted highest-order terms are functions of η. In a rigorous adjustment of the highest-order terms, one would first assign practical extreme values to (hwo - 1) and the parameters. One would then obtain exact solutions to equations (4.3) and (4.4) for these conditions. Next, one would evaluate the series of type (4.17), (4.18) through one order less than the highest order to be used. Finally, one would subtract the next to highest order evaluations from the exact solutions. The differences obtained would give the adjusted highest-order terms. The series with adjusted highest-order terms could be checked against the exact solutions with moderate combinations of (hwo - 1) and the parameters.

The method that was actually used to adjust the highest-order terms was an adjustment on these terms as numerically calculated (table I). The method was simply to multiply each highest-order term, ${}_iG_j$ and ${}_iF_j$, by a constant. The constants were determined by matching the wall derivatives, $G'(0)$ and $F''(0)$, with those obtained from exact solutions of equations (4.3), (4.4). The possibility of using this method comes from the fact that for a given parameter, the curves of related universal functions shown in figure 2 have approximately similar shapes. As an example, in figure 2, we can compare the curves for ${}_1F_1'$, ${}_1F_2'$, and ${}_1F_3'$, which are all connected with the parameter g_1,

and we can see that the shapes are roughly similar. We can surmise that terms of higher order connected with g_1 would continue to have approximately this shape, or, if the higher-order terms would be lumped into one remainder term, it would have this same approximate shape. So it seems likely that a reasonable remainder term can be obtained if ${}_1F_3$ is multiplied by a constant. The "rigorous" method previously described is exact at the conditions for which it is calculated, but it, like the method actually used, can only be a good approximation at conditions other than those for which calculated.

1. Exact Solutions

In order to adjust the highest-order terms in our expansions, we need exact solutions of equations (4.3) and (4.4) with boundary conditions (4.5). In obtaining exact solutions, selected values of the gas property parameters were inserted into (4.3) and (4.4). A selected value of hwo was inserted in equations (4.3) and (4.4) and also in the boundary condition (4.5b). In programming the solutions, the third-order equation (4.4) was split into two second-order equations in order to use an existing subroutine (the procedure described in sect. A). The procedure used in starting the solutions was to guess the "missing" initial (wall) conditions, $F''(0)$ and $G'(0)$, and run the case through to the outer boundary ($\eta = 10$). A systematic procedure of incrementing or decrementing the initial conditions, $F''(0)$ and $G'(0)$, was used until the two outer boundary conditions (4.5c) were "bracketed" (last two trials gave values greater and less than the correct outer boundary conditions). Both initial conditions were adjusted simultaneously as it was found that while the equations interact with each other, equation (4.3) is most sensitive to $G'(0)$ while (4.4) is most sensitive to $F''(0)$. It frequently happened that one equation found a bracket while the other was still incrementing toward its bracket. Further incrementing of the second initial condition sometimes resulted in the first equation being thrown out of its bracket as a result of interaction, so this contingency had to be provided for. When both outer boundary conditions were bracketed, linear interpolation was used to determine a new pair of initial conditions, and the increment sizes

on the initial conditions were reduced. The above described incrementing procedure was then repeated until new brackets were found. This procedure was repeated as many times as needed with decreasing increment sizes until the outer boundary conditions were satisfied to six decimal places. The simple routine described above was programmed, so that the final answers were obtained as output in one machine operation. Most cases were handled in from 20 to 40 "passes." One "pass" required about 15 seconds of machine time.

2. Adjustment of Highest-Order Terms Not Involving σ_{wo}

In order to determine the values of the constant multipliers of highest-order terms not involving σ_{wo} in the G and F expansions, a number of exact solutions of equations (4.3) and (4.4) were obtained with $\sigma_{wo} = \sigma_o = 0.75$ (or $\sigma_1 = 0$ which is the same thing) and with hwo = 0 (an extreme value).* The extreme values of the gas property parameters were selected as values that would more than encompass the range of air properties for a flight Mach number of about 10. The values selected are

$$m_1 = -1.0 \qquad m_2 = 1.5 \qquad m_3 = -4.5 \tag{5.2a}$$

$$g_1 = -1.5 \qquad g_2 = 2.0 \qquad g_3 = -6.0 \tag{5.2b}$$

$$\delta_1 = 1.0 \qquad \delta_2 = -0.4 \qquad \delta_3 = 1.2 \tag{5.2c}$$

These parameter values were inserted in equations (4.3), (4.4) in various combinations, and the constant multipliers in the series expansions were determined by matching the wall derivatives, G'(0) and F"(0), between the exact and the series solutions. With the values of the constants inserted in the G and F expansions, comparisons were made between exact solutions and series expansion solutions using hwo values of 0.2 and 0.5. This gave comparisons of answers under conditions different from those used to determine the constants. The values of the constants determined by the procedure described are shown below in section D, equations (5.3) and (5.4).

*The use of hwo = 0 does not imply that we expect to physically realize this value in a boundary layer. It is a mathematically convenient extreme value to be inserted into equations (4.3), (4.4), and (4.5b).

The comparisons of the values of the wall derivatives obtained from the exact solutions and those obtained from the expansions are summarized in table 5-1. Parameters listed for a case have the values listed in table 5-1; the other parameters for a particular case are zero. The underlined values of the calculated wall derivatives are those for which the constants were adjusted to make the calculated derivatives equal to the exact derivatives (at hwo = 0). In some cases both $G'(0)$ and $F''(0)$ are underlined which means that two constants were determined. In other cases (e.g., case m_1), only one derivative is underlined which means that only one constant was available for adjustment.

Of the cases listed in table 5-1, the largest error seen is 2.5 percent which occurred in two cases, case m_3 and case m_2,m_3. Most of the comparisons are within 1 percent. This is thought to be a very satisfactory check. It was not necessary to check the cases at hwo = 1. All cases check exactly at this wall enthalpy value since the perturbation is made around hwo = 1.

The cases listed in table 5-1 were compared throughout the boundary layer. Plots were made comparing the exact and calculated values of $G(\eta)$ and $F'(\eta)$ for the cases listed at the three hwo values used. As examples, the plots of several cases are shown in figure 4. The δ_1 case (figs. 4(a-c)) shows the effect of one parameter. Two constants were determined for this case (underlined in table 5-1). It is seen that the agreement throughout the boundary layer is very good. The g_1,δ_1 case plotted in figures 4(d-f) shows the effects of a combination of two parameters. Two constants were determined for this case, and again the agreement is very good.

In the latter part of table 5-1 are shown some cases involving combinations of parameters for which no constants were determined (not underlined). Constants could have been determined for these cases, but the terms associated with the constants were dropped in order to keep the G and F series to reasonable lengths. The terms were dropped on the basis of having only a small effect on the wall derivatives. This could be expected to possibly produce larger errors away from the

TABLE 5-1.- COMPARISON OF WALL DERIVATIVES WITH WALL PRANDTL NUMBER FIXED

$\sigma_{wo} = \sigma_o = 0.75$

Case	hwo	G'(0) exact	G'(0) calculated	F"(0) exact	F"(0) calculated
No parameters	0	0.48311958	0.48311958	0.92768005	0.92768005
	.2	.38649566	.38649566		
	.5	.24155979	.24155979		
m_1	0	.42406524	.42406524		
	.2	.3448	.3419		
	.5	.2223	.2202		
g_1	0	.48344722	.48344722	.84740213	.84740213
	.2	.3867	.3868	.8550	.8656
	.5	.2417	.2417	.8712	.8792
δ_1	0	.44046294	.44046293	.55851770	.55851769
	.2	.3603	.3601	.6375	.6373
	.5	.2319	.2317	.7506	.7505
m_1, g_1	0	.42336594	.42336594	.8493	.8474
	.2	.3444	.3417	.8565	.8656
	.5	.2221	.2202	.8719	.8792
m_1, δ_1	0	.38565439	.38565439	.5610	.5585
	.2	.3209	.3177	.6394	.6373
	.5	.2132	.2109	.7515	.7505
g_1, δ_1	0	.43670058	.43670058	.47699449	.47699448
	.2	.3584	.3582	.5646	.5751
	.5	.2314	.2314	.6942	.7023
m_1, g_1, δ_1	0	.3808	.3809	.4786	.4770
	.2	.3183	.3153	.5663	.5751
	.5	.2125	.2104	.6954	.7023
m_2	0	.40931441	.40931442	.92768005	.92768005
	.2	.3425	.3487	.92768005	.92768005
	.5	.2285	.2323	.92768005	.92768005
g_2	0	.4855	.4831	.82045738	.82045737
	.2	.3879	.3865	.8453	.8591
	.5	.2420	.2416	.8865	.9009

TABLE 5-1.- COMPARISON OF WALL DERIVATIVES WITH WALL PRANDTL NUMBER FIXED - Continued

$\sigma_{wo} = \sigma_o = 0.75$

Case	hwo	G'(0) exact	G'(0) calculated	F"(0) exact	F"(0) calculated
δ_2	0	0.47884533	0.47884533	0.87701186	0.87701185
	.2	.3843	.3843	.8953	.8953
	.5	.2410	.2410	.9151	.9150
m_2, g_1	0	.4083	.4096	.8479	.8474
	.2	.3420	.3490	.8553	.8656
	.5	.2284	.2325	.8712	.8792
m_2, δ_1	0	.3727	.3667	.5651	.5585
	.2	.3191	.3223	.6415	.6373
	.5	.2193	.2225	.7517	.7505
m_1, g_2	0	.4251	.4241	.8238	.8205
	.2	.3455	.3419	.8476	.8591
	.5	.2225	.2202	.8873	.9009
m_1, δ_2	0	.4200	.4198	.8762	.8770
	.2	.3427	.3397	.8949	.8953
	.5	.2217	.2196	.9150	.9150
g_1, g_2	0	.4847	.4834	.80711266	.80711265
	.2	.3875	.3868	.8220	.8312
	.5	.2419	.2417	.8514	.8608
g_1, δ_2	0	.4788	.4792	.79642401	.79642401
	.2	.3844	.3846	.8225	.8330
	.5	.2411	.2412	.8585	.8665
g_2, δ_1	0	.4382	.4405	.45526707	.45526706
	.2	.3598	.3601	.5581	.5707
	.5	.2320	.2317	.7105	.7242
δ_1, δ_2	0	.4341	.4362	.49905285	.49905287
	.2	.3574	.3579	.6012	.6004
	.5	.2312	.2312	.7371	.7367
g_3	0	.4878	.4831	.79256256	.79256256
	.2	.3890	.3865	.8389	.8585
	.5	.2420	.2416	.8996	.9108

TABLE 5-1.- COMPARISON OF WALL DERIVATIVES WITH WALL PRANDTL NUMBER FIXED - Continued

$\sigma_{wo} = \sigma_o = 0.75$

Case	hwo	G'(0) exact	G'(0) calculated	F''(0) exact	F''(0) calculated
δ_3	0	0.48025849	0.48025849	0.88735514	0.88735513
	.2	.3853	.3853	.9071	.9070
	.5	.2414	.2414	.9227	.9226
m_3	0	.39541513	.39541513	.92768005	.92768005
	.2	.3422	.3506	.92768005	.92768005
	.5	.2333	.2361	.92768005	.92768005
g_1, g_2 $g_1 = -0.75$ $g_2 = 1.5$	0	.4847	.4834	.8227	.8239
	.2	.3875	.3867	.8401	.8452
	.5	.2419	.2416	.8711	.8771
δ_1, δ_2 $\delta_1 = 0.8$ $\delta_2 = -0.4$	0	.4446	.4458	.5805	.5796
	.2	.3634	.3637	.6632	.6624
	.5	.2333	.2333	.7737	.7734
δ_1, δ_3	0.	.4362	.4376	.51167579	.51167579
	.2	.3587	.3589	.6146	.6140
	.5	.2316	.2316	.7452	.7450
g_1, g_2, g_3	0	.4866	.4834	.77085440	.77085441
	.2	.3886	.3868	.7956	.8046
	.5	.2422	.2417	.8400	.8503
g_1, g_3	0	.4861	.4834	.79130612	.79130612
	.2	.3881	.3868	.8174	.8288
	.5	.2420	.2417	.8573	.8673
g_2, g_3	0	.4880	.4831	.76129792	.76129793
	.2	.3894	.3865	.7990	.8148
	.5	.2424	.2416	.8664	.8864
δ_2, δ_3	0	.4759	.4760	.8361	.8367
	.2	.3831	.3831	.8746	.8746
	.5	.2409	.2408	.9101	.9100
$\delta_1, \delta_2, \delta_3$	0	.42960535	.42960535	.4508	.4522
	.2	.3558	.3559	.5779	.5771
	.5	.2310	.2310	.7317	.7312

TABLE 5-1.- COMPARISON OF WALL DERIVATIVES WITH WALL PRANDTL NUMBER FIXED - Continued

$\sigma_{wo} = \sigma_o = 0.75$

Case	hwo	G'(0) exact	G'(0) calculated	F"(0) exact	F"(0) calculated
g_1, g_2, g_3, δ_1	0	0.4366	0.4367	0.4066	0.4044
	.2	.3588	.3582	.5089	.5161
	.5	.2317	.2314	.6643	.6739
$g_1, g_2, g_3, \delta_1, \delta_2, \delta_3$	0	.4228	.4258	.2950	.2978
	.2	.3535	.3540	.4482	.4557
	.5	.2308	.2307	.6453	.6546
m_1, m_2	0	.39103534	.39103534	.92768005	.92768005
	.2	.3236	.3208		
	.5	.2148	.2135		
m_1, m_2 $m_1 = -0.5$ $m_2 = 1.0$	0	.4102	.4047		
	.2	.3392	.3378		
	.5	.2235	.2239		
m_1, m_3	0	.38199193	.38199194		
	.2	.3226	.3210		
	.5	.2174	.2161		
m_2, m_3	0	.36654293	.36654292		
	.2	.3169	.3246		
	.5	.2221	.2276		
m_1, m_2, m_3	0	.36381192	.36381191		
	.2	.3079	.3053		
	.5	.2107	.2099		
$m_1, m_2, m_3, g_1, g_2, g_3, \delta_1, \delta_2, \delta_3$	0	.3096	.3098	.2921	.2978
	.2	.2777	.2746	.4503	.4557
	.5	.2005	.1994	.6475	.6546
m_1, m_2, g_1, g_2	0	.3901	.3903	.8114	.8071
	.2	.3231	.3206	.8250	.8312
	.5	.2147	.2135	.8528	.8608
m_1, m_2, m_3, δ_1	0	.3307	.3254	.5683	.5585
	.2	.2865	.2811	.6437	.6373
	.5	.2021	.2006	.7527	.7505

TABLE 5-1.- COMPARISON OF WALL DERIVATIVES WITH WALL PRANDTL NUMBER FIXED - Concluded

$\sigma_{wo} = \sigma_o = 0.75$

Case	hwo	G'(0) exact	G'(0) calculated	F"(0) exact	F"(0) calculated
$m_1, m_2, m_3, \delta_1, \delta_2$	0	0.3253	0.3211	0.5071	0.4991
	.2	.2839	.2789	.6065	.6004
	.5	.2015	.2001	.7391	.7367
$m_1, m_2, m_3, \delta_1, \delta_2, \delta_3$	0	.3211	.3145	.4553	.4522
	.2	.2824	.2769	.5820	.5771
	.5	.2013	.1999	.7336	.7312
$m_1, m_2, g_1, g_2, \delta_1$	0	.3486	.3478	.4426	.4407
	.2	.2977	.2942	.5367	.5427
	.5	.2052	.2037	.6771	.6843
$m_1, m_2, m_3, g_1, g_2, g_3, \delta_1$	0	.3224	.3206	.4125	.4044
	.2	.2827	.2787	.5141	.5161
	.5	.2014	.2001	.6669	.6739
$m_1, m_2, m_3, g_1, g_2, g_3, \delta_1, \delta_2$	0	.3153	.3163	.3477	.3446
	.2	.2796	.2765	.4758	.4790
	.5	.2007	.1996	.6531	.6601

Note: Quantities underlined have been matched with exact values by adjustment of constants. Parameters not listed are zero for a particular case.

Values of parameters unless otherwise shown:

$m_1 = -1.0$ $\quad g_1 = -1.5$ $\quad \delta_1 = 1.0$

$m_2 = 1.5$ $\quad g_2 = 2.0$ $\quad \delta_2 = -0.4$

$m_3 = -4.5$ $\quad g_3 = -6.0$ $\quad \delta_3 = 1.2$

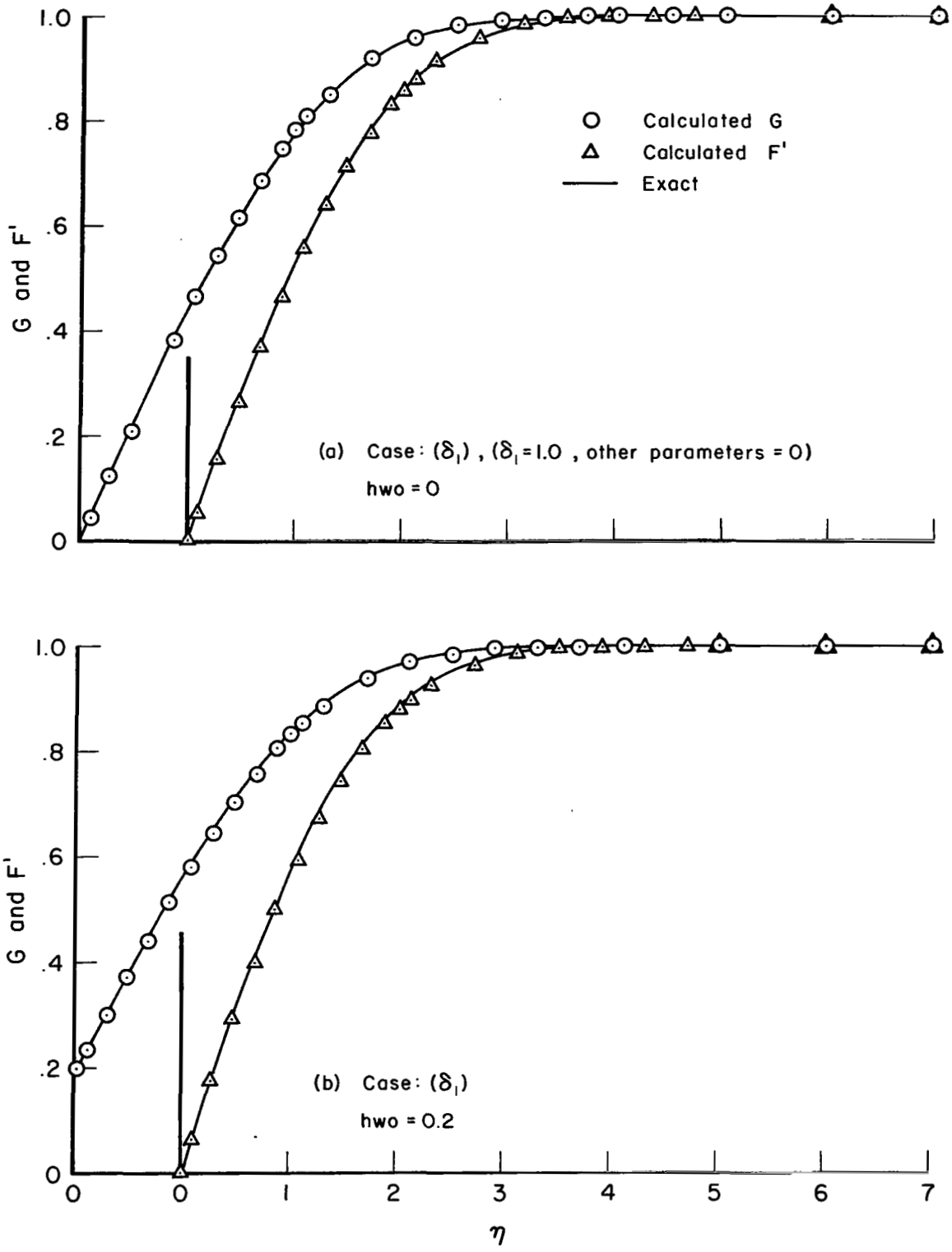


Figure 4.- Comparison of exact and calculated values of G and F'; $\sigma_{wo} = \sigma_o = 0.75$.

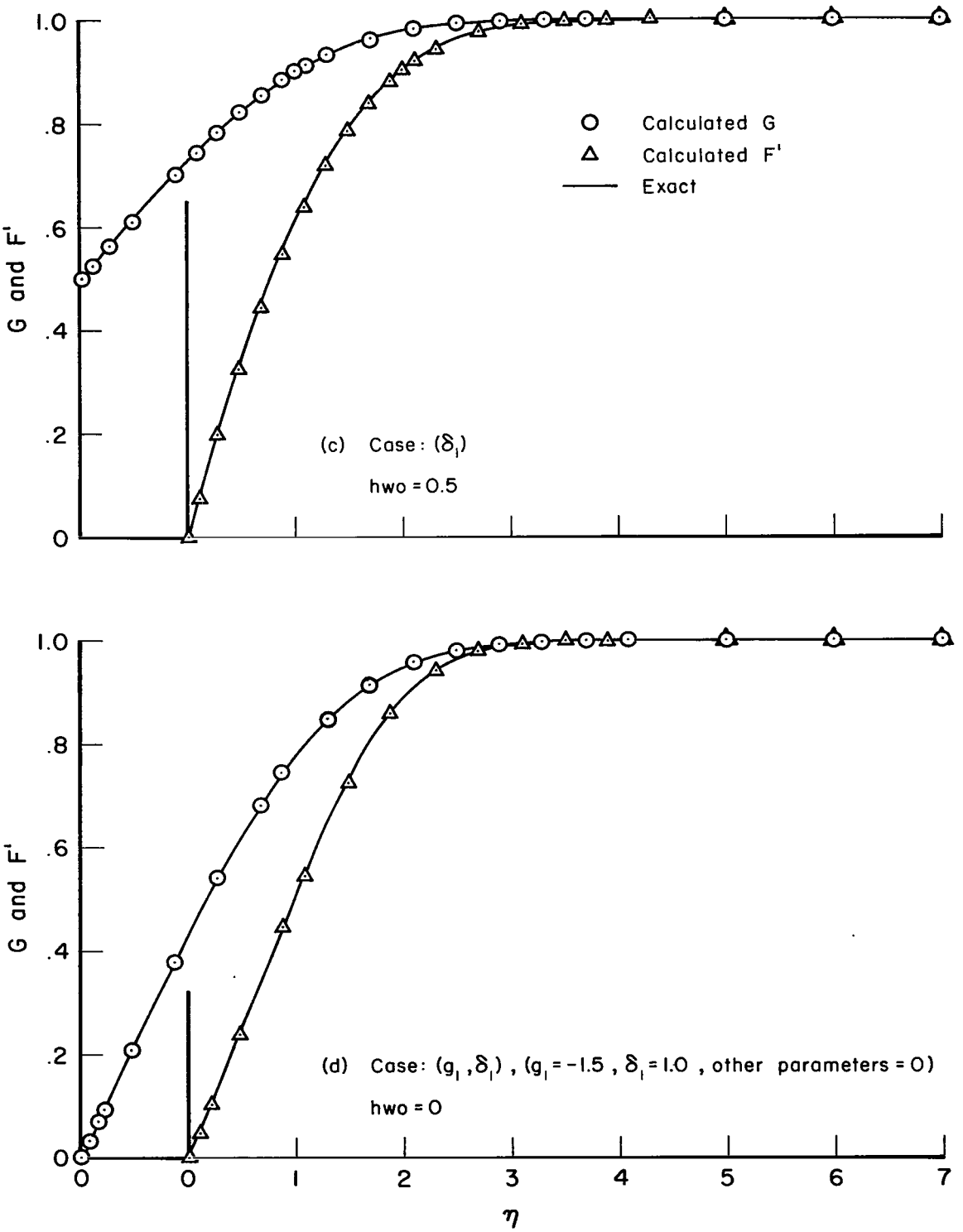


Figure 4.- Continued.

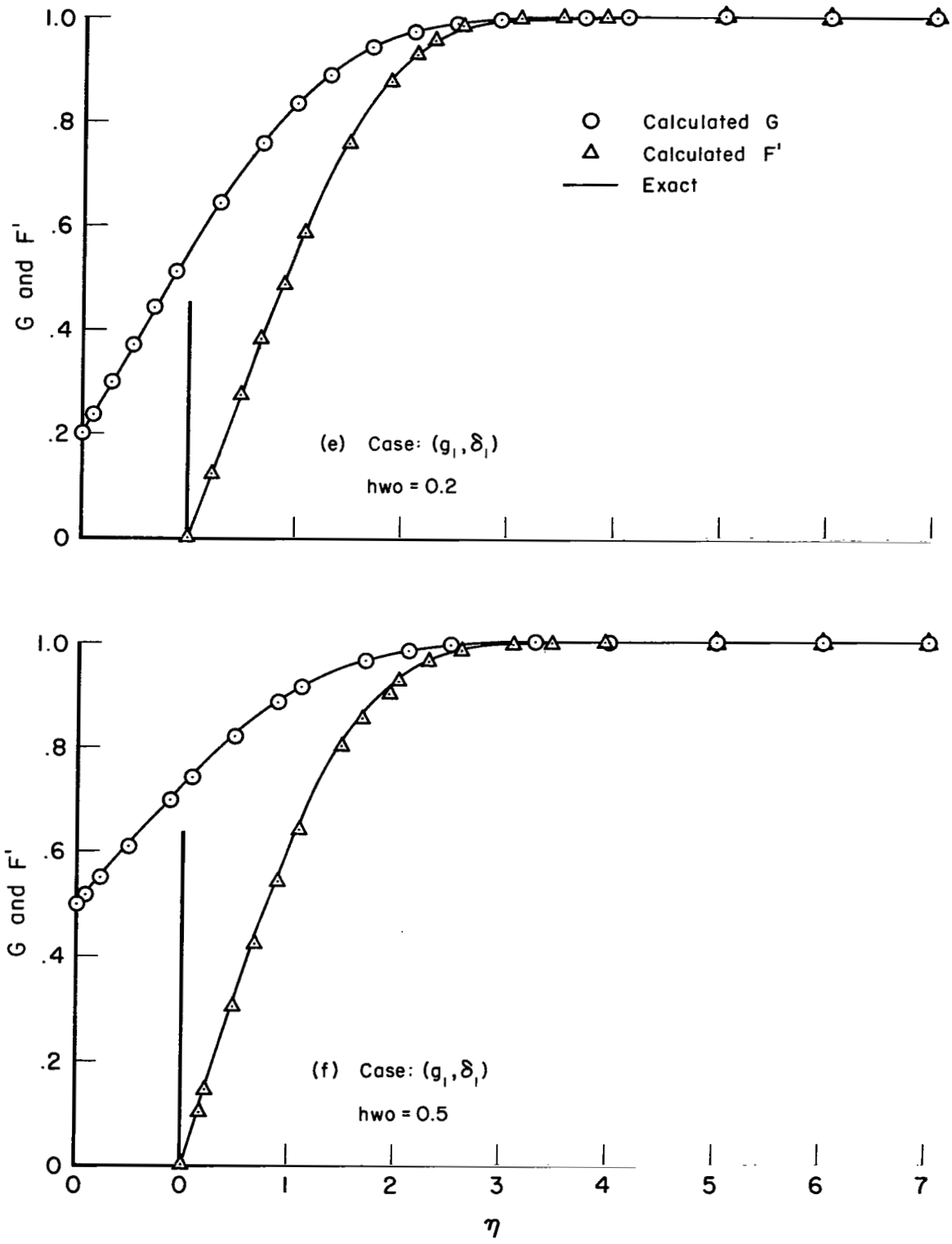


Figure 4.- Continued.

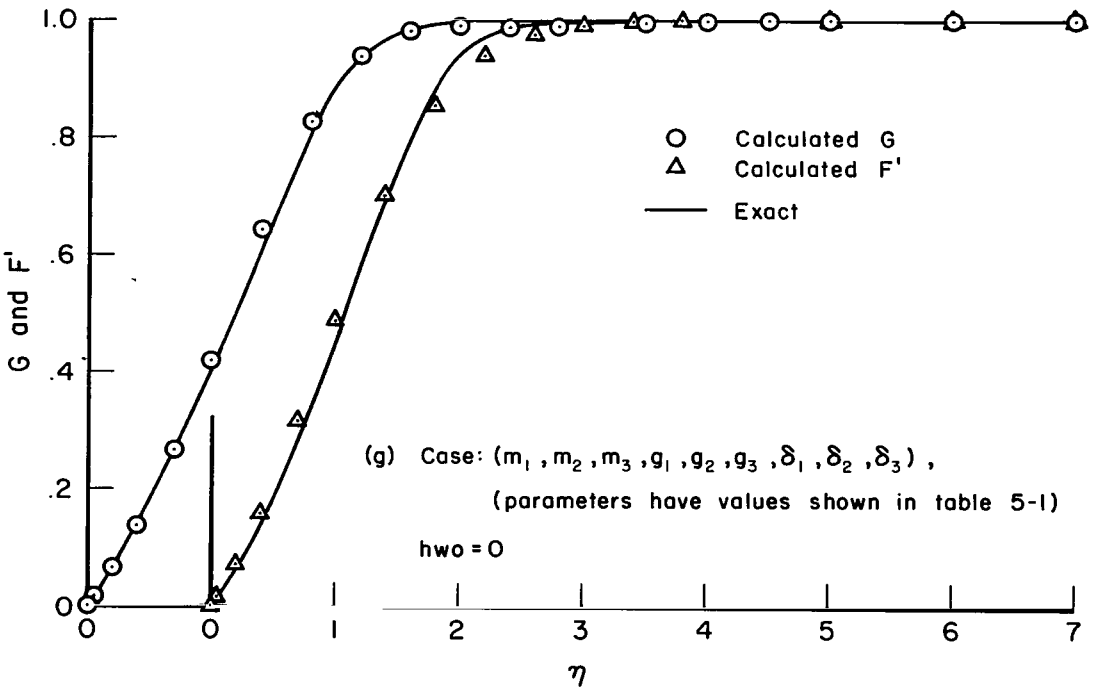


Figure 4.- Concluded.

wall because, as seen in figure 2, the universal functions reach their asymptotes at different rates and at different values of η. The most extreme case listed in table 5-1 is $m_1,m_2,m_3,g_1,g_2,g_3,\delta_1,\delta_2,\delta_3$ in which all the parameters have their extreme values. This case, with hwo = 0, is shown in figure 4(g). It is seen that where the curve bends to reach its asymptote there is an error in F' of about 4 percent. This case, with all parameters simultaneously taking their extreme values, and hwo = 0 is actually more severe than we contemplate for applications of the method (examples in ch. 7) and more extreme than we will have for flight Mach numbers up to 10. The errors at a distance from the wall that we will have in applications are typified in figures 7 and 11 (cases described in ch. 7). The errors, due to dropping small terms and to truncations of the G and F series (see fig. 1), go to zero as hwo approaches unity. The δ_1 and g_1,δ_1 cases as plotted in figure 4 are typical of the accuracy

obtained in cases for which constants have been determined. On the whole, the checks shown in table 5-1, a few examples of which are shown plotted in figure 4, are considered to be quite satisfactory.

3. Method of Adjusting σ_{wo} Terms

The method of determining the constant multipliers of the highest-order σ_{wo} terms in the G and F expansions is essentially the same as that described in the previous subsection for the terms not involving σ_{wo}. A number of exact solutions of equations (4.3) and (4.4) were obtained with various combinations of gas-property parameters, various values of the wall Prandtl number, σ_{wo}, and with $h_{wo} = 0$. The purpose was to determine which terms in the series for G and F have any significant dependency on σ_{wo}. These terms with the constants evaluated are seen below in equations (5.3) and (5.4).

The constants were adjusted to match the $G'(0)$ and $F''(0)$ of the exact solutions for a value of $\sigma_{wo} = 0.70$, $h_{wo} = 0$, and with the gas-property parameter values shown in equations (5.2). Then comparisons were made with exact solutions by considering σ_{wo} values of 0.65 and 0.80 for $h_{wo} = 0$. A few cases were also checked with $h_{wo} = 0.2$ and 0.5. Comparisons of the wall derivatives for these determinations and checks are shown in table 5-2. As in table 5-1, the underlined values represent wall derivatives for which a constant was determined to match the value of the derivative given by the exact solution.

Table 5-2 gives a survey of the effects of σ_{wo} on the wall derivatives $G'(0)$ and $F''(0)$. The table shows that our first-order perturbation of the effect of σ_{wo} is very good. In the cases listed for which a constant was determined at $\sigma_{wo} = 0.70$ (underlined), the errors in the same derivative at σ_{wo} values of 0.65 and 0.80 are due to departures from linearity of the effects of σ_{wo}. We have set limits for the values of σ_{wo} at $0.65 \leq \sigma_{wo} \leq 0.80$, because 0.65 and 0.80 are the values we used in checking against exact solutions. The results shown in table 5-2 suggest that these limits could be extended somewhat. However, these limits encompass the range of σ_{wo} values of

TABLE 5-2.- COMPARISON OF WALL DERIVATIVES WITH VALUES OF THE WALL PRANDTL NUMBER

σ_{wo} = 0.65, 0.70, 0.75, 0.80

Case	σ_{wo}	hwo	G'(0) exact	G'(0) calculated	F''(0) exact	F''(0) calculated
No parameters	0.65	0	0.4572	0.4579	0.92768005	0.92768005
	.65	.2	.3658	.3663		
	.65	.5	.2286	.2289		
	.70	0	.47049304	.47049303		
	.75	0	.4831	.4831		
	.75	.2	.3865	.3865		
	.75	.5	.2416	.2416		
	.80	0	.4952	.4957		
m_1	.65		.4017	.4023		
	.70		.41316916	.41316915		
	.75		.4241	.4241		
	.80		.4345	.4350		
m_2	.65		.3878	.3883		
	.70		.39882710	.39882709		
	.75		.4093	.4093		
	.80		.4193	.4198		
m_3	.65		.3747	.3752		
	.70		.38531153	.38531153		
	.75		.3954	.3954		
	.80		.4051	.4055		
m_1, m_2	.65		.3706	.3711		
	.70		.38107903	.38107901		
	.75		.3910	.3910		
	.80		.4005	.4010		
m_1, m_3	.65		.3621	.3626		
	.70		.37228429	.37228429		
	.75		.3820	.3820		
	.80		.3913	.3917		
m_2, m_3	.65		.3475	.3480		
	.70		.35726044	.35726043		
	.75		.3665	.3665		
	.80		.3754	.3758		

TABLE 5-2.- COMPARISON OF WALL DERIVATIVES WITH VALUES OF THE WALL PRANDTL NUMBER - Continued

σ_{wo} = 0.65, 0.70, 0.75, 0.80

Case	σ_{wo}	hwo	G'(0) exact	G'(0) calculated	F''(0) exact	F''(0) calculated
m_1, m_2, m_3	0.65	0	0.3449	0.3454	0.92768005	0.92768005
	.70		.35461786	.35461783		
	.75		.3638	.3638		
	.80		.3726	.3730		
g_1	.65		.4577	.4584	.8515	.8514
	.70		.47091208	.47091206	.84941414	.84941413
	.75		.4834	.4834	.8474	.8474
	.80		.4954	.4960	.8455	.8454
g_2	.65		.4595	.4579	.8248	.8247
	.70		.4728	.4705	.82255814	.82255813
	.75		.4855	.4831	.8205	.8205
	.80		.4976	.4957	.8185	.8184
g_3	.65		.4616	.4579	.7971	.7969
	.70		.4750	.4705	.79474424	.79474424
	.75		.4878	.4831	.7926	.7926
	.80		.5001	.4957	.7905	.7904
g_1, g_2	.65		.4589	.4584	.8127	.8126
	.70		.4721	.4709	.80984925	.80984925
	.75		.4847	.4834	.8071	.8071
	.80		.4967	.4960	.8045	.8044
g_1, g_3	.65		.4601	.4584	.7970	.7969
	.70		.4734	.4709	.79408304	.79408304
	.75		.4861	.4834	.7913	.7913
	.80		.4982	.4960	.7887	.7885
g_2, g_3	.65		.4618	.4579	.7676	.7674
	.70		.4752	.4705	.76435109	.76435110
	.75		.4880	.4831	.7613	.7613
	.80		.5002	.4957	.7584	.7582
g_1, g_2, g_3	.65		.4606	.4584	.7775	.7773
	.70		.4739	.4709	.77408756	.77408757
	.75		.4866	.4834	.7709	.7709
	.80		.4986	.4960	.7678	.7676

TABLE 5-2.- COMPARISON OF WALL DERIVATIVES WITH VALUES OF THE WALL PRANDTL NUMBER - Continued

σ_{wo} = 0.65, 0.70, 0.75, 0.80

Case	σ_{wo}	hwo	G'(0) exact	G'(0) calculated	F''(0) exact	F''(0) calculated
δ_1	0.65	0	0.4156	0.4163	0.5470	0.5475
	.65	.2	.3403	.3405	.6288	.6285
	.65	.5	.2192	.2194	.7454	.7450
	.70	0	.42835946	.42835945	.55301907	.55301908
	.75	0	.4405	.4405	.5585	.5585
	.75	.2	.3603	.3601	.6375	.6373
	.75	.5	.2319	.2317	.7506	.7505
	.80	0	.4521	.4526	.5637	.5640
δ_2	.65	0	.4531	.4536	.8752	.8753
	.70	0	.4663	.4662	.87613608	.87613607
	.75	0	.4788	.4788	.8770	.8770
	.80	0	.4909	.4915	.8778	.8779
δ_3	.65	0	.4544	.4550	.8857	.8858
	.70	0	.4677	.4676	.88657984	.88657983
	.75	0	.4803	.4803	.8874	.8874
	.80	0	.4923	.4929	.8881	.8881
δ_1,δ_2	.65	0	.4094	.4120	.4851	.4856
	.70	0	.4221	.4241	.49234293	.49234296
	.75	0	.4341	.4362	.4991	.4991
	.80	0	.4457	.4483	.5053	.5058
δ_1,δ_3	.65	0	.4115	.4134	.4981	.4986
	.70	0	.4241	.4255	.50514187	.50514190
	.75	0	.4362	.4376	.5117	.5117
	.80	0	.4478	.4497	.5177	.5182
m_1,g_1	.65	0	.4014	.4018	.8536	.8514
	.65	.2	.3264	.3238	.8604	.8688
	.65	.5	.2105	.2085	.8749	.8812
	.75	0	.4234	.4234	.8493	.8474
	.75	.2	.3444	.3417	.8565	.8656
	.75	.5	.2221	.2202	.8719	.8792
m_1,g_1,δ_1	.65	0	.3600	.3603	.4715	.4700
	.65	.2	.3012	.2981	.5614	.5695
	.65	.5	.2012	.1989	.6932	.6988
	.75	0	.3808	.3809	.4786	.4770
	.75	.2	.3183	.3153	.5663	.5751
	.75	.5	.2125	.2104	.6954	.7023
	.80	0	.3905	.3911	.4817	.4805

TABLE 5-2.- COMPARISON OF WALL DERIVATIVES WITH VALUES OF THE WALL PRANDTL NUMBER - Concluded

σ_{wo} = 0.65, 0.70, 0.75, 0.80

Case	σ_{wo}	hwo	G'(0) exact	G'(0) calculated	F"(0) exact	F"(0) calculated
$m_1,m_2,m_3,$ $g_1,g_2,g_3,$ $\delta_1,\delta_2,\delta_3$	0.65	0	0.2926	0.2926	0.2826	0.2888
	.65	.2	.2628	.2595	.4451	.4497
	.65	.5	.1898	.1885	.6457	.6514
	.75	0	.3096	.3098	.2921	.2978
	.75	.2	.2777	.2746	.4503	.4557
	.75	.5	.2005	.1994	.6475	.6546
	.80	0	.3176	.3183	.2962	.3023
g_1,δ_1	.65	0	.4125	.4127	.4700	.4700
	.75	0	.4367	.4367	.4770	.4770
$g_1,g_2,$ g_3,δ_1	.65	0	.4123	.4127	.4021	.3999
	.75	0	.4366	.4367	.4066	.4044
	.80	0	.4479	.4487	.4085	.4067
$g_1,g_2,g_3,$ $\delta_1,\delta_2,\delta_3$	.65	0	.3987	.4018	.2856	.2888
	.75	0	.4228	.4258	.2950	.2978
	.80	0	.4341	.4379	.2991	.3023
$m_1,m_2,m_3,$ $g_1,g_2,g_3,$ δ_1	.65	0	.3054	.3035	.4083	.3999
	.75	0	.3224	.3206	.4125	.4044
	.80	0	.3304	.3292	.4143	.4067
m_1,δ_1	.65	0	.3642	.3649	.5493	.5475
	.75	0	.3857	.3857	.5610	.5585
$\delta_1,\delta_2,\delta_3$	.65	0	.4049	.4054	.4345	.4367
	.75	0	.4296	.4296	.4508	.4522

Note: Quantities underlined have been matched with exact values at σ_{wo} = 0.70 by adjustment of constants. Parameters not listed are zero for a particular case. Values of parameters are as listed in table 5-1.

gases in which we might have interest, so these limits can be considered as practical values. Almost all errors in table 5-2 at $\sigma_{wo} \neq 0.75$ are less than 1 percent. The largest errors are in case $m_1, m_2, m_3, g_1, g_2, g_3, \delta_1, \delta_2, \delta_3$ (2.2 percent at $\sigma_{wo} = 0.65$ and 2.1 percent at $\sigma_{wo} = 0.80$) and in case $m_1, m_2, m_3, g_1, g_2, g_3, \delta_1$ (2.1 percent at $\sigma_{wo} = 0.65$ and 1.8 percent at $\sigma_{wo} = 0.80$). Even these errors are not mainly due to the σ_{wo} perturbation because almost the same errors exist for these cases with $\sigma_{wo} = \sigma_o = 0.75$.

As was done with the data for table 5-1, plots were made of the table 5-2 data comparing exact and computed values of $G(\eta)$ and $F'(\eta)$ throughout the boundary layer. The plots of several cases are shown in figure 5. At hwo = 0 and $\sigma_{wo} = 0.70$ are shown plots of three cases for which constants were determined: the m_1 case (fig. 5(a)), the g_1 case (fig. 5(b)), and the δ_1 case (fig. 5(c)). It is seen that the agreements shown are very good. In the m_1 case the $F'(\eta)$ curve is not shown since both the exact and calculated solutions are precisely the Homann function. In figures 5(d) and (e) are shown the

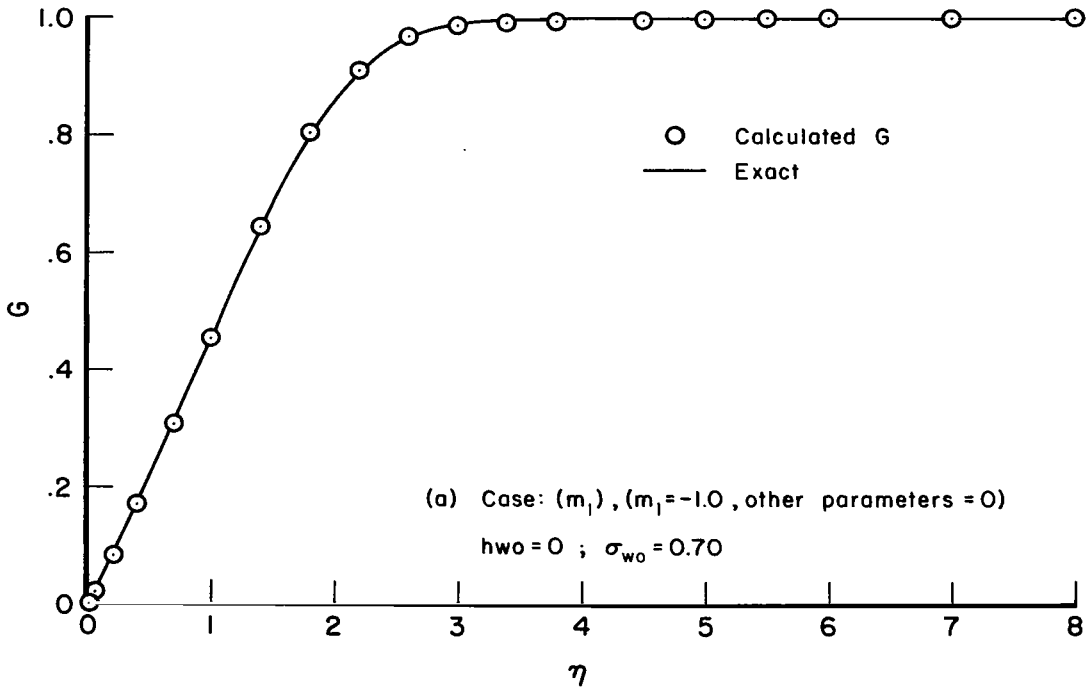


Figure 5.- Comparison of exact and calculated values of G and F' with variation of σ_{wo}.

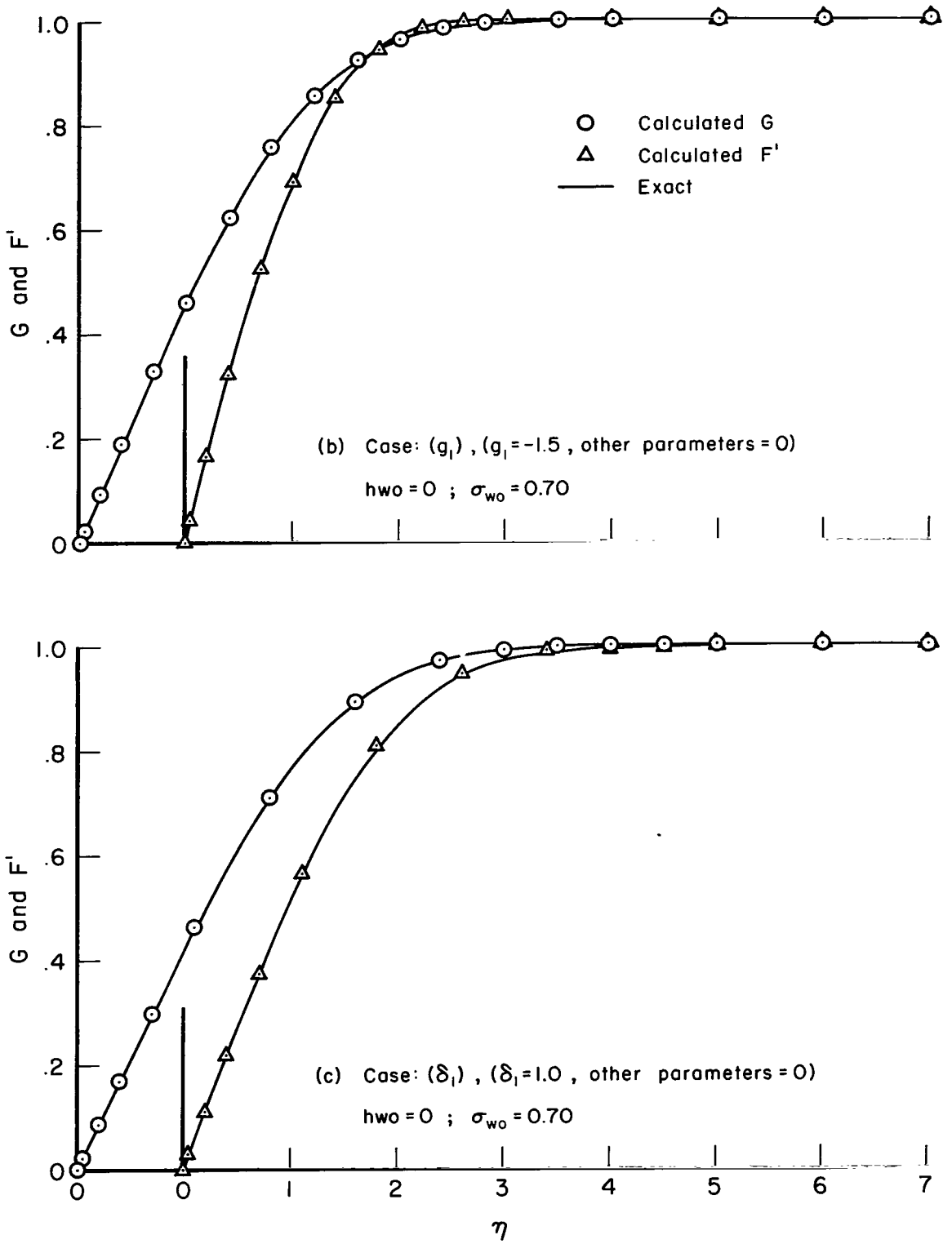


Figure 5.- Continued.

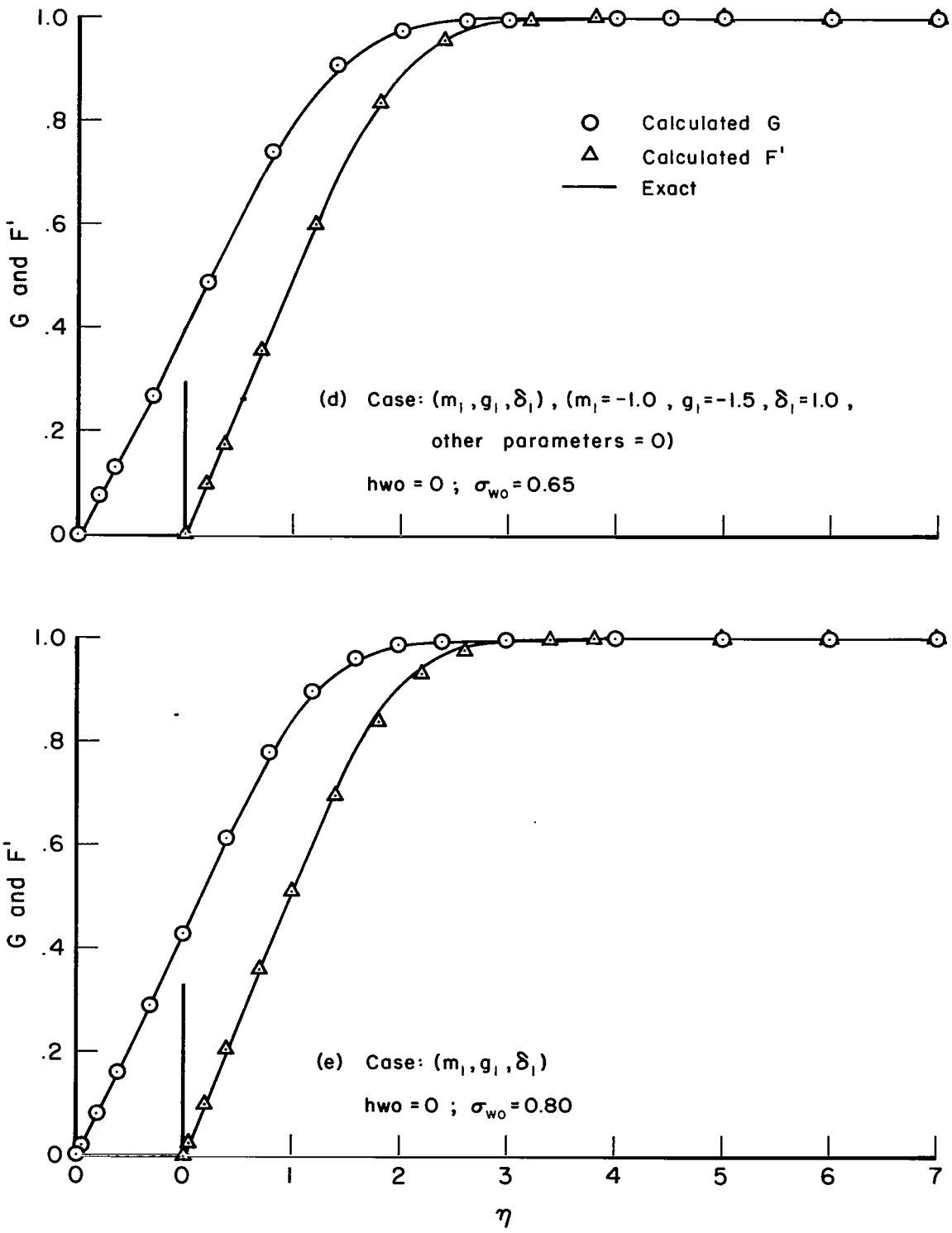


Figure 5.- Concluded.

m_1, g_1, δ_1 case with hwo = 0 and with the σ_{wo} values 0.65 and 0.80. It is seen that the agreement is still good for these σ_{wo} values even though the constants were determined for $\sigma_{wo} = 0.70$. In figure 5(e) some error can be seen where the F' curve bends over to its asymptote. This is due mainly to the dropping of small terms from the F series (all terms that would involve m_i were dropped from this series; see fig. 1(b)). In the most extreme case with all parameters taking their extreme values, the errors at the different σ_{wo} values are about the same as shown in figure 4(g) for $\sigma_{wo} = \sigma_o = 0.75$. In all cases the errors decrease to zero as hwo approaches unity. On the whole, the checks obtained for $\sigma_{wo} \neq \sigma_o$, as shown in table 5-2 and typified by the plots in figure 5, are considered to be very satisfactory.

D. FINAL SERIES FORM OF SOLUTIONS

With the constant multipliers given their numerical values, and with $\sigma_o = 0.75$, we write the series for G and F in final form:

$$
\begin{aligned}
G = 1 &+ [{}_1G_1 + 1.02317726(\sigma_{wo} - 0.75){}_2G_1](hwo - 1) + \{[1 + 0.30901832(\sigma_{wo} \\
&- 0.75)](m_1){}_1G_2 + [1 - 2.42190802(\sigma_{wo} - 0.75)](g_1){}_2G_2 \\
&+ [1 + 0.29146138(\sigma_{wo} - 0.75)](\delta_1){}_3G_2\} \frac{(hwo - 1)^2}{2} + \{0.52193773(m_1^2){}_1G_3 \\
&+ 0.24383382(m_1g_1){}_2G_3 + 0.40200776(g_1^2){}_3G_3 + 0.70153983(g_1\delta_1){}_4G_3 \\
&+ 1.3280404(\delta_1^2){}_5G_3 + 0.60407205(m_1\delta_1){}_6G_3 + [0.60669608 \\
&+ 0.35169961(\sigma_{wo} - 0.75)](m_2){}_7G_3 + 1.0182513(\delta_2){}_8G_3\} \frac{(hwo - 1)^3}{6} \\
&+ \{[0.61202131 + 0.35211282(\sigma_{wo} - 0.75)](m_3){}_1G_4 + [0.22173048 \\
&+ 0.13045010(\sigma_{wo} - 0.75)](m_1m_2){}_2G_4 + 1.01272912(\delta_3){}_3G_4\} \frac{(hwo - 1)^4}{24} \\
&+ \{[0.22546289 + 0.13187495(\sigma_{wo} - 0.75)](m_1m_3){}_1G_5\} \frac{(hwo - 1)^5}{120} \\
&+ \{[0.26063754 + 0.15291739(\sigma_{wo} - 0.75)](m_2m_3){}_1G_6\} \frac{(hwo - 1)^6}{720} \\
&+ \{[0.07210034 + 0.04275391(\sigma_{wo} - 0.75)](m_1m_2m_3){}_1G_7 \\
&+ 621.9886774(\delta_1\delta_2\delta_3){}_2G_7\} \frac{(hwo - 1)^7}{5040} \qquad (5.3)
\end{aligned}
$$

$$
\begin{aligned}
F = F_o &+ \{[1 + 0.21023047(\sigma_{wo} - 0.75)](g_1)_1F_1 \\
&+ [1 - 0.32104920(\sigma_{wo} - 0.75)](\delta_1)_2F_1\}(hwo - 1) \\
&+ \{(g_1^2)_1F_2 + (g_1\delta_1)_2F_2 + (\delta_1^2)_3F_2 \\
&+ [0.53415792 + 0.20930977(\sigma_{wo} - 0.75)](g_2)_4F_2 \\
&+ [1.0079927 - 0.34845525(\sigma_{wo} - 0.75)](\delta_2)_5F_2\} \frac{(hwo - 1)^2}{2} \\
&+ \{0.41447166(g_1^3)_1F_3 + 1.3743224(\delta_1^3)_2F_3 \\
&+ 0.51720661[(g_1^2\delta_1)_3F_3 + (g_1\delta_1^2)_4F_3] \\
&+ [0.14873715 + 0.06116220(\sigma_{wo} - 0.75)](g_1g_2)_5F_3 \\
&+ 0.62726245(g_1\delta_2)_6F_3 + 0.33313689(g_2\delta_1)_7F_3 \\
&+ [1.5242090 - 1.16268794(\sigma_{wo} - 0.75)](\delta_1\delta_2)_8F_3 \\
&+ [0.54408038 + 0.17570032(\sigma_{wo} - 0.75)](g_3)_9F_3 \\
&+ [1.0055625 - 0.38666553(\sigma_{wo} - 0.75)](\delta_3)_{10}F_3\} \frac{(hwo - 1)^3}{6} \\
&+ \{[0.15281523 + 0.05479625(\sigma_{wo} - 0.75)](g_1g_3)_1F_4 \\
&+ [1.4873902 - 1.18676619(\sigma_{wo} - 0.75)](\delta_1\delta_3)_2F_4\} \frac{(hwo - 1)^4}{24} \\
&+ \{[0.19177545 + 0.06207212(\sigma_{wo} - 0.75)](g_2g_3)_1F_5\} \frac{(hwo - 1)^5}{120} \\
&+ \{[0.039081277 + 0.01338353(\sigma_{wo} - 0.75)](g_1g_2g_3)_1F_6\} \frac{(hwo - 1)^6}{720}
\end{aligned}
\tag{5.4}
$$

Equations (5.3), (5.4) are the working formulas giving solutions to equations (4.3), (4.4) with boundary conditions (4.5). The agreement shown in tables 5-1 and 5-2, and typified by the plots in figures 4 and 5, forms the justification for representing solutions to equations (4.3), (4.4) in the form of the two series given, with the parameters of the problem serving as coefficients in the series. The limitation on the validity of the series solutions is that the gas properties should be representable by third-degree polynomials (discussed further in sect. E, below, and appendix D). The values of the gas-property parameters, m_i, g_i, δ_i, can be outside the range of values listed in equations (5.2), but the actual terms in the gas-property

polynomials (e.g., $(m_i/i!)(G - 1)^i$) are limited in magnitude and should have the sign for which the adjustments of constants were made. The magnitude limitation of the gas-property polynomial terms is obtained by using the values of the parameters listed in equation (5.2) and setting $G = 0$ in the gas-property polynomials. This limitation, then, corresponds to the values of the gas-property polynomial terms for which the constants were adjusted. (These limitations are tabulated in table E-1.) The value of the wall Prandtl number is expected to be in the range $0.65 \leq \sigma_{wo} \leq 0.80$. Normally, hwo should not be greater than unity (heated wall case), as the constants were adjusted for hwo - 1 = -1 (see sect. C.2 above). Slight departures from the limitations may be allowable if the highest-order terms in equations (5.3), (5.4) are small.

For given values of hwo and the gas-property parameters (σ_{wo}, m_1, m_2, m_3, g_1, g_2, g_3, δ_1, δ_2, δ_3) the values of $G(\eta)$ and $F(\eta)$ can be obtained through the boundary layer if the tabulated values given in table I for ${}_iG_j$ and ${}_iF_j$ are substituted into formulas (5.3), (5.4). The values of the derivatives with respect to η through the boundary layer, $G'(\eta)$, $G''(\eta)$, $F'(\eta)$, $F''(\eta)$, $F'''(\eta)$, can also be obtained from (5.3), (5.4) by use of the tabulated derivatives of the universal functions. Equations (5.3), (5.4) (and the derivatives) have been machine programmed for easy solution, the inputs being hwo and the gas-property parameters, with the universal functions being input from the permanent binary tape on which they are stored.

The values of the wall derivatives, $G'(0)$ and $F''(0)$ are of special interest. Using equations (5.3) and (5.4) for these derivatives, and substituting the values of the wall derivatives of the universal functions, ${}_iG_j'(0)$ and ${}_iF_j''(0)$, we obtain:

$$G'(0) = [-0.48311958 - 0.2525308(\sigma_{wo} - 0.75)](hwo - 1) + \{[0.22399449 + 0.0692184(\sigma_{wo} - 0.75)]m_1 + [-0.0010063678 + 0.00243733(\sigma_{wo} - 0.75)]g_1 + [-0.071784467 - 0.0209224(\sigma_{wo} - 0.75)]\delta_1\} \frac{(hwo - 1)^2}{2} + \{-0.31765746m_1^2 + 0.0041077599m_1g_1 + 0.0011390400g_1^2 - 0.016360000g_1\delta_1 + 0.040586460\delta_1^2 + 0.025474740m_1\delta_1 + [0.29522068 + 0.1711384(\sigma_{wo} - 0.75)]m_2 - 0.064113744\delta_2\} \frac{(hwo - 1)^3}{6} + \{[0.46775706 + 0.26911360(\sigma_{wo} - 0.75)]m_3 + [-0.65240416 - 0.38382720(\sigma_{wo} - 0.75)]m_1m_2 - 0.05722180\delta_3\} \frac{(hwo - 1)^4}{24} + \{[-1.2168304 - 0.71173333(\sigma_{wo} - 0.75)]m_1m_3\} \frac{(hwo - 1)^5}{120} + \{[-4.79284906 - 2.81198933(\sigma_{wo} - 0.75)]m_2m_3\} \frac{(hwo - 1)^6}{720} + \{[22.462033 + 13.319488(\sigma_{wo} - 0.75)]m_1m_2m_3 - 39.083520\delta_1\delta_2\delta_3\} \frac{(hwo - 1)^7}{5040} \quad (5.5)$$

$$F''(0) = 0.92768005 + \{[-0.12760662 - 0.02682680(\sigma_{wo} - 0.75)]g_1 + [0.34254127 - 0.10997260(\sigma_{wo} - 0.75)]\delta_1\}(hwo - 1) + \{0.23707484g_1^2 - 0.0047159187g_1\delta_1 - 0.041946936\delta_1^2 + [-0.10722267 - 0.04201520(\sigma_{wo} - 0.75)]g_2 + [0.25334096 - 0.087578(\sigma_{wo} - 0.75)]\delta_2\} \frac{(hwo - 1)^2}{2} + \{-0.27658167g_1^3 + 0.0087819495g_1^2\delta_1 - 0.0059559956g_1\delta_1^2 + 0.033885660\delta_1^3 + [0.13386640 + 0.05504720(\sigma_{wo} - 0.75)]g_1g_2 + 0.00309920g_1\delta_2 - 0.011916120g_2\delta_1 + [-0.13194960 + 0.10065300(\sigma_{wo} - 0.75)]\delta_1\delta_2 + [-0.13511749 - 0.04363360(\sigma_{wo} - 0.75)]g_3 + [0.20162455 - 0.077530(\sigma_{wo} - 0.75)]\delta_3\} \frac{(hwo - 1)^3}{6} + \{[0.21072395 + 0.07556107(\sigma_{wo} - 0.75)]g_1g_3 + [-0.13033960 + 0.10399600(\sigma_{wo} - 0.75)]\delta_1\delta_3\} \frac{(hwo - 1)^4}{24} + \{[0.75958040 + 0.24585400(\sigma_{wo} - 0.75)]g_2g_3\} \frac{(hwo - 1)^5}{120} + \{[-2.24481120 - 0.768744(\sigma_{wo} - 0.75)]g_1g_2g_3\} \frac{(hwo - 1)^6}{720} \quad (5.6)$$

The machine programming of the derivatives of equations (5.3) and (5.4) includes the programming of equations (5.5) and (5.6).

The constants in equations (5.3), (5.4), (5.5), and (5.6) are shown as delivered by the computing machine with eight "significant figures" or decimal places. These are the numbers used by the computing machine for the solution of equations (5.3), (5.4), (5.5), and (5.6) with the universal functions being input from the permanent binary tape. For hand computations, the constants can be rounded off inasmuch as the universal functions are rounded off to five decimal places in table I, and it appears certain that more decimal places are not meaningful (see discussion in sect. B above). A further rounding off may be used when one considers the accuracy of results required and the accuracy shown in tables 5-1 and 5-2.

E. REPRESENTATION OF THE GAS PROPERTIES BY POLYNOMIALS

To solve a problem by means of the formulas given in this chapter, it is necessary to know the gas-property parameters, m_i, g_i, δ_i, preferably through the third order. As an aid to our discussion we will rewrite equations (3.1) in the G notation as third-degree polynomials.

$$m(G) = \frac{\rho\mu/\sigma}{(\rho\mu/\sigma)_{SO}} = 1 + m_1(G - 1) + \frac{m_2}{2}(G - 1)^2 + \frac{m_3}{6}(G - 1)^3 \qquad (5.7a)$$

$$g(G) = \frac{\rho\mu}{(\rho\mu)_{SO}} = 1 + g_1(G - 1) + \frac{g_2}{2}(G - 1)^2 + \frac{g_3}{6}(G - 1)^3 \qquad (5.7b)$$

$$\delta(G) = \frac{\rho_{SO}}{\rho} = 1 + \delta_1(G - 1) + \frac{\delta_2}{2}(G - 1)^2 + \frac{\delta_3}{6}(G - 1)^3 \qquad (5.7c)$$

There are several ways of determining the gas-property parameters. If the gas-property data are known only numerically, a least squares fit can be used to determine the parameters (the coefficients in the third-degree polynomials). Another method is to force each polynomial to pass through four selected points (one point being (1, 1)). This involves the simultaneous solutions of three linear algebraic equations.

If the gas properties are known analytically, it may be desirable to expand them into Taylor series to the third order about (1, 1). This will determine the first two coefficients in each polynomial. The third coefficient can be determined if the third-order Taylor coefficient is adjusted to give the best fit to the data (truncating the Taylor series). Whichever method is used to determine the gas-property parameters, the polynomials obtained should be plotted to compare with the data they are supposed to represent. Up to flight Mach number values of about 5 in the earth's atmosphere, the gas properties can be very well represented by third-degree polynomials. At Mach numbers around 10, the representation can be considered to be fairly good. Some examples of this are given in chapter 7.

The values of the gas-property parameters listed in equations (5.2) and used in obtaining the results shown in tables 5-1 and 5-2 have been given signs which correspond to realistic gas properties. It is possible to use a coefficient with a sign opposite to the signs listed in equations (5.2) to pass a polynomial through a discrete number of points, but this is not desirable (and probably would not give the best representation of the gas properties). It is preferable to determine another polynomial by a different choice of points. In table E-1 are listed the allowable ranges of all the gas-property parameters (which depend on the value of hwo). The use of parameter values outside of the allowable range (on either end) should only be permissible when the highest-order term involving the parameter is small relative to the other terms in the series for G and F.

As has been indicated above, there are several reasonable ways of fitting polynomials to the gas properties and they can result in somewhat different combinations of coefficients in the polynomials. The question arises as to what effects the use of different polynomials can have on the final answers. This is discussed in appendix D.

The results in appendix D show that our calculated answers are not very sensitive to the shapes of the gas-property polynomials used. Also the answers are not much affected by small errors in the wall values of the several gas properties. The matching of the gas

properties by polynomials will normally be much closer than the spread of the polynomials imposed in appendix D. The results in appendix D indicate that any errors in final answers due to imperfect representation of gas properties should not be more than a few percent. Equations (5.3), (5.4), (5.5), (5.6) have large terms which are independent of the gas properties. Also, the sensitivity of final answers to the polynomials selected falls to zero as hwo approaches unity.

F. APPROXIMATE EXTENSION OF THE SERIES SOLUTION TO HIGHER ORDERS

The method we have used to form our series-type solutions as given in equations (5.3) and (5.4) can be extended to higher orders of (hwo - 1) if desired. Such solutions would allow the gas properties to be represented by polynomials of higher than third degree, and this would provide better representation of the gas properties at high free-stream Mach numbers.

It is possible to obtain approximations to the higher-order terms without solving new equations for higher-order universal functions. It is seen in figure 2 that the universal functions which are connected with the same type of parameter (m_i, g_i, δ_i and combinations) have approximately the same shapes. As an example we can see in figure 2 that the curves for ${}_1F_1'$, ${}_1F_2'$, ${}_1F_3'$, ${}_4F_2'$, ${}_9F_3'$ (which are connected with the parameters g_1, g_1^2, g_1^3, g_2, g_3, respectively) all have roughly the same shape; similarly, the curves for ${}_5F_3'$, ${}_1F_4'$, ${}_1F_5'$, ${}_1F_6'$ (connected respectively with parameters g_1g_2, g_1g_3, g_2g_3, $g_1g_2g_3$) have about the same shapes (in some cases with the sign reversed); this type of approximation for the other functions shown in figure 2 is seen to be fairly consistent. So if, for instance, we wanted to approximate a new function ${}_aF_4$ (connected with the parameter g_4), we would guess that it would have the same shape as ${}_9F_3$ (parameter g_3), and we would take

$$ {}_aF_4 = (C){}_9F_3 $$

where C is a constant to be determined by matching the wall value of F"(0) for a selected value of g_4 (and hwo) against the wall value obtained from exact solutions to equations (4.3) and (4.4). This is just the method that we used in truncating the highest-order terms for which we determined the universal functions. The author checked this type of approximation for a number of cases. An example case used the parameter values $m_3 = -4.5$, $\sigma_{wo} = \sigma_o$, the other parameters equal to zero, and hwo assumed the three values shown in table 5-1. In the approximation, ${}_1G_4$ (parameter m_3) was taken as

$$ {}_1G_4 = (C)_7G_3 $$

(${}_7G_3$ has the parameter m_2). Values of G for this case were determined from the approximate ${}_1G_4$, and the exact ${}_1G_4$, and were plotted against η through the boundary layer. Differences in the plots in this and in other cases checked were scarcely discernible. The wall derivatives, of course, were exactly matched.

The approximate method outlined for extending the series solutions to higher orders may occasionally be of value when it appears that higher-order polynomials are needed to fit the gas properties. However, an indefinite extension of this kind would seem generally not to be warranted in order to obtain a good polynomial fit to the gas properties at high free-stream Mach numbers (say 10 and above), since our method of series solutions does not account for time-dependent chemical effects, and these assume increased importance at the higher free-stream Mach numbers.

CHAPTER 6

INVERSE TRANSFORMATIONS AND DETERMINATION OF QUANTITIES OF INTEREST

In chapter 2, we transformed the boundary-layer equations from an x,y to a ξ,η coordinate system for the axisymmetric case. In chapter 3 we obtained the stagnation-point equations (3.12), (3.13), and in chapter 4 we rewrote these equations in the form (4.3), (4.4). These equations are valid in the immediate vicinity of the stagnation point since F and G are the zero order of the f and h expansions (3.2) and (3.3), the first terms neglected being of order ξ^2. In this chapter we will specialize the coordinate transformations (2.39) and (2.40) to the vicinity of the stagnation point. Then we will invert the specialized transformations, since in many problems one will want to recover the physical coordinates. Also, there are certain physical quantities and coefficients which are normally defined and written in terms of untransformed variables. These will be set up in terms of η and the dependent variables, F,G.

A. INVERSE TRANSFORMATIONS

1. Transformation for x

In the vicinity of the stagnation point we have

$$u_e \approx \left(\frac{du_e}{dx}\right)_{(x=0)} x$$

$$r(x) \approx x$$

$$(\rho_w \mu_w)(0) = (\rho\mu)_{wo}$$

We substitute the values into equation (2.39), integrate, and obtain

$$\xi = \left[\frac{1}{4}(\rho\mu)_{wo}\left(\frac{du_e}{dx}\right)_{(x=0)}\right]^{1/4} x \qquad (6.1)$$

Using equation (3.4) we can write

$$u_e \sim \left(\frac{du_e}{dx}\right)_{(x=0)} x \approx \sqrt{2H_{st}}\, a_1 \xi \tag{6.2}$$

Then for the inverse transformation from ξ to x in the vicinity of the stagnation point we have the following forms:

$$x = \left[\frac{1}{4}(\rho\mu)_{wo}\left(\frac{du_e}{dx}\right)_{(x=0)}\right]^{-1/4}\xi = \left[\frac{1}{4}(\rho\mu)_{so}g_{wo}\left(\frac{du_e}{dx}\right)_{(x=0)}\right]^{-1/4}\xi$$

$$= \left[\frac{1}{4}(\rho\mu)_{so}g_{wo}a_1\sqrt{2H_{st}}\right]^{-1/3}\xi \tag{6.3}$$

The relationship between $(du_e/dx)_{(x=0)}$ and a_1 can be obtained by equating the second and third bracketed quantities in (6.3).

$$\left(\frac{du_e}{dx}\right)_{(x=0)} = \left[\frac{1}{4}(\rho\mu)_{so}g_{wo}a_1^4(2H_{st})^2\right]^{1/3} \tag{6.4}$$

By equating the first and third bracketed quantities in equation (6.3) and by using (6.4) we can obtain the following additional relations between $(du_e/dx)_{(x=0)}$ and a_1:

$$\left[\frac{(\rho\mu)_{so}g_{wo}}{\sqrt{2}\,a_1^2(2H_{st})}\right]^{1/3} = \sqrt{\frac{(\rho\mu)_{wo}}{2(du_e/dx)_{(x=0)}}} = \sqrt{\frac{(\rho\mu)_{so}g_{wo}}{2(du_e/dx)_{(x=0)}}} \tag{6.5}$$

$$\left[\frac{1}{4}(\rho\mu)_{so}g_{wo}a_1\sqrt{2H_{st}}\right]^{2/3} = \frac{1}{2}\sqrt{(\rho\mu)_{wo}\left(\frac{du_e}{dx}\right)_{(x=0)}} = \frac{1}{2}\sqrt{(\rho\mu)_{so}g_{wo}\left(\frac{du_e}{dx}\right)_{(x=0)}} \tag{6.6}$$

These relationships will be useful in the work to follow.

2. Transformation for y

We can specialize equation (2.40) for the vicinity of the stagnation point by using (6.1) and we obtain

$$\eta = \sqrt{\frac{2(du_e/dx)_{(x=o)}}{(\rho\mu)_{so}g_{wo}}} \int_o^y \rho(0,y_1)dy_1 \qquad (6.7)$$

The first term neglected in equation (6.7) is in x^2.

We can easily invert equation (6.7) to obtain

$$\frac{y}{\sqrt{\mu_{so}g_{wo}/2\rho_{so}(du_e/dx)_{(x=o)}}} = \frac{y}{(1/\rho_{so})\sqrt{(\rho\mu)_{wo}/2(du_e/dx)_{(x=o)}}}$$

$$= \frac{y}{(1/\rho_{so})[(\rho\mu)_{so}g_{wo}/\sqrt{2}\ a_1^2(2H_{st})]^{1/3}} = \int_o^\eta \delta(G)d\eta_1 \qquad (6.8)$$

The denominator in the third term of equation (6.8) was obtained from the first term of (6.5). Equation (6.8) can be integrated numerically after the profile of $G(\eta)$ has been determined. This integral has been machine programmed to be calculated at the same time the G profile is obtained. Equation (6.8) is the general working formula for the transformation from η back to y. Two special cases will be described next.

3. Determination of y for Almost Perfect and Perfect Gases

We will define an almost perfect gas as one having a linear relationship between the reciprocal of the dimensionless density, $\delta(G)$, and the dimensionless enthalpy, G, over the range of interest. This means that $\delta_2 = \delta_3 = 0$ and

$$\delta_{AP} = 1 + \delta_1(G - 1) = (1 - \delta_1) + \delta_1 G \qquad (6.9)$$

For perfect gases (thermally and calorically perfect), $\delta_1 = 1$ and we have

$$\delta_P = G \tag{6.10}$$

For the almost perfect gas case, the integral in equation (6.8) becomes

$$\int_o^\eta \delta_{AP}(G)d\eta_1 = (1 - \delta_1)\eta + \delta_1 \int_o^\eta G(\eta_1)d\eta_1 \tag{6.11}$$

We can substitute for G in equation (6.11) its series expansion form (eq. (5.3)). Then we will have a summation of integrals of the universal functions, ${}_iG_j$. These are our universal integrals, ${}_iI_j$, defined in equation (5.1), tabulated in table I, and shown plotted in figure 3.

We now specialize equation (6.8) for the calculation of y (in the vicinity of the stagnation point) for almost perfect gases as follows:

$$\frac{y_{AP}}{\sqrt{\mu_{so}g_{wo}/2\rho_{so}(du_e/dx)_{(x=o)}}}$$

$$= \frac{y_{AP}}{(1/\rho_{so})\sqrt{(\rho\mu)_{wo}/2(du_e/dx)_{(x=o)}}} = \frac{y_{AP}}{(1/\rho_{so})[(\rho\mu)_{so}g_{wo}/\sqrt{2}\ a_1{}^2(2H_{st})]^{1/3}}$$

$$= \eta + (\delta_1)[{}_1I_1 + 1.02317726(\sigma_{wo} - 0.75){}_2I_1](hwo - 1) + (\delta_1)\{[1$$

$$+ 0.30901832(\sigma_{wo} - 0.75)](m_1){}_1I_2 + [1 - 2.42190802(\sigma_{wo} - 0.75)](g_1){}_2I_2$$

$$+ [1 + 0.29146138(\sigma_{wo} - 0.75)](\delta_1){}_3I_2\} \frac{(hwo - 1)^2}{2}$$

$$+ (\delta_1)\{0.52193773(m_1{}^2){}_1I_3 + 0.24383382(m_1g_1){}_2I_3 + 0.40200776(g_1{}^2){}_3I_3$$

$$+ 0.70153983(g_1\delta_1){}_4I_3 + 1.3280404(\delta_1{}^2){}_5I_3 + 0.60407205(m_1\delta_1){}_6I_3$$

$$+ [0.60669608 + 0.35169961(\sigma_{wo} - 0.75)](m_2){}_7I_3\} \frac{(hwo - 1)^3}{6}$$

$$+ (\delta_1)\{[0.61202131 + 0.35211282(\sigma_{wo} - 0.75)](m_3){}_1I_4 + [0.22173048$$

$$+ 0.13045010(\sigma_{wo} - 0.75)](m_1m_2){}_2I_4\} \frac{(hwo - 1)^4}{24} + (\delta_1)\{[0.22546289$$

$$+ 0.13187495(\sigma_{wo} - 0.75)](m_1m_3){}_1I_5\} \frac{(hwo - 1)^5}{120} + (\delta_1)\{[0.26063754$$

$$+ 0.15291739(\sigma_{wo} - 0.75)](m_2m_3){}_1I_6\} \frac{(hwo - 1)^6}{720} + (\delta_1)\{[0.07210034$$

$$+ 0.04275391(\sigma_{wo} - 0.75)](m_1m_2m_3){}_1I_7\} \frac{(hwo - 1)^7}{5040} \tag{6.12}$$

For the perfect gas case we calculate y_p by inserting $\delta_1 = 1$ in equation (6.12) which is a working formula that can be used in place of equation (6.8) for the transformation from η back to y for almost perfect and perfect gases. This equation has also been machine programmed with the ${}_iI_j$ inputs taken from the permanent binary tape. For hand computations the constants in the equation can be rounded off (see ch. 5, sect. D). Equation (6.12) contains terms corresponding to equation (5.3) (except for terms with δ_2 or δ_3 which are zero). Selected parameter values can make some of the terms either zero or negligibly small; approximate values can often be hand calculated using only two terms. Except for the first term, the sum of the terms on the right side of equation (6.12) usually reaches its asymptote at a small η value. In figure 8(a) are shown curves obtained by solving equation (6.12). The establishment of linearity between y and η means that the sum of terms mentioned has reached its asymptote, and compressibility does not then affect the y,η relationship.

B. PROFILES OF ρv, ρ, AND v NEAR THE STAGNATION POINT

We have a general formula for the ρv profile (eq. (2.18)) in the variables τ and η. For the axisymmetric case, we substitute $\epsilon = 1$ and $\tau = \xi^4$ into equation (2.18) and obtain

$$\rho v = -(\xi f_\xi + 2f)\frac{\sqrt{2}\,(\rho\mu)_s(\xi)g_w(\xi)u_e r}{4\xi^2} - \frac{\sqrt{2}\,\xi^2 f_\eta \eta_x}{r} \qquad (6.13)$$

From equation (2.40) we know that η is even in x and $\eta_x = 0$ at the stagnation point. We specialize (6.13) to the vicinity of the stagnation point using $r \approx x$, $u_e \approx (du_e/dx)_{(x=0)}x$, and equations (6.3) and (6.6). We obtain

$$\frac{-\rho v}{2\sqrt{2}[(1/4)(\rho\mu)_{so}g_{wo}a_1\sqrt{2H_{st}}]^{2/3}} = \frac{-\rho v}{\sqrt{2(\rho\mu)_{wo}(du_e/dx)_{(x=0)}}}$$

$$= \frac{-\rho v}{\sqrt{2(\rho\mu)_{so}g_{wo}(du_e/dx)_{(x=0)}}} = F \qquad (6.14)$$

Equation (6.14) is valid in the vicinity of the stagnation point, the first term neglected being a term in ξ^2.

For the density profile we return to the definition of δ (eq. (2.24a)) specialized to the stagnation point

$$\rho = \frac{\rho_{so}}{\delta(G)} \tag{6.15}$$

For almost perfect gases we have from equation (6.9):

$$\rho_{AP} = \frac{\rho_{so}}{(1 - \delta_1) + \delta_1 G} \tag{6.16}$$

For perfect gases we set $\delta_1 = 1$ in equation (6.16) and obtain ρ_P.

For the transverse velocity profile near the stagnation point we have from equation (6.15):

$$v = \frac{(\rho v)\delta(G)}{\rho_{so}} \tag{6.17}$$

Using equation (6.14) we can write (6.17) as

$$\frac{-v}{(2\sqrt{2}/\rho_{so})[(1/4)(\rho\mu)_{so} g_{wo} a_1 \sqrt{2H_{st}}]^{2/3}}$$

$$= \frac{-v}{(1/\rho_{so})\sqrt{2(\rho\mu)_{wo}(du_e/dx)_{(x=o)}}}$$

$$= \frac{-v}{\sqrt{(2\mu_{so} g_{wo}/\rho_{so})(du_e/dx)_{(x=o)}}} = F\delta(G) \tag{6.18}$$

The quantity, v, is, of course, even in ξ, the first neglected term in equation (6.18) being of order ξ^2. For almost perfect gases we substitute equation (6.9) into (6.18) and have for the right-hand side of equation (6.18):

$$F\delta_{AP} = [(1 - \delta_1) + \delta_1 G]F \tag{6.19}$$

from which we calculate v_{AP}. For perfect gases we set $\delta_1 = 1$ in equation (6.19) and obtain v_P.

C. QUANTITIES AND COEFFICIENTS OF INTEREST

1. Displacement Thickness, δ^*

We write the definition of displacement thickness:

$$\delta^* = \int_o^\infty \left(1 - \frac{\rho u}{\rho_e u_e}\right) dy_1 \tag{6.20}$$

In the variable, η, we have:

$$\delta^* = \int_o^\infty \left(1 - \frac{\rho}{\rho_e} f_\eta\right) \frac{d\eta_1}{(\partial\eta/\partial y)_{(x=\text{constant})}} \tag{6.21}$$

We now specialize equation (6.21) to the vicinity of the stagnation point, making use of equation (6.7). We recall that at the stagnation point, $\rho_e = \rho_{so}$. Also we change the upper limit of integration from infinity to Δ, where Δ is a value of η at, or beyond, the outer edge of the boundary layer, such that $f_\eta = 1$ and $\rho = \rho_e$ to within the accuracy desired. We obtain:

$$\delta^* = \frac{1}{\rho_{so}} \sqrt{\frac{(\rho\mu)_{so} g_{wo}}{2(du_e/dx)_{(x=o)}}} \left[\int_o^\Delta \delta(G) d\eta_1 - F(\Delta)\right] \tag{6.22}$$

From equation (6.8) we see that the first term of equation (6.22) is $y(\Delta)$, which is as it should be. The form of equation (6.22) is a subtraction of two quantities after integration of the two terms in equation (6.20). With accurate integration routines this should be as accurate as the procedure of first subtracting in (6.20) and then integrating.

Analogous to our forms of equation (6.8) we write

$$\frac{\delta^*}{\sqrt{\mu_{so} g_{wo}/2\rho_{so}(du_e/dx)_{(x=o)}}}$$

$$= \frac{\delta^*}{(1/\rho_{so})\sqrt{(\rho\mu)_{wo}/2(du_e/dx)_{(x=o)}}}$$

$$= \frac{\delta^*}{(1/\rho_{so})[(\rho\mu)_{so} g_{wo}/\sqrt{2}\, a_1^2 (2H_{st})]^{1/3}} = \int_o^\Delta \delta(G) d\eta_1 - F(\Delta) \tag{6.23}$$

Equation (6.23) is our general working formula for δ^* in the vicinity of the stagnation point, the first neglected term being a term in ξ^2. This equation has been machine programmed using $\Delta = 10$. For hand computations, one obtains $F(\Delta)$ using equation (5.4) and tabulated values of ${}_iF_j$ in table I. For the integral term, one uses table I and equation (5.3) to obtain the G profile, thence the δ profile; the integration can be performed by numerical means.

As in our determination of y, we can specialize the calculation of δ^* for the almost perfect and perfect gas cases (see eqs. (6.9) and (6.10)). For the almost perfect gas case, the right side of equation (6.23) becomes

$$\int_0^{\Delta} \delta_{AP}(G)d\eta_1 - F(\Delta) = (1 - \delta_1)\Delta + \delta_1 \int_0^{\Delta} G d\eta_1 - F(\Delta) \qquad (6.24)$$

The integral on the right side of equation (6.24) is a summation of our ${}_iI_j$ integrals (5.1), using the form of equation (5.3). Using the value, $\Delta = 10$, and the ${}_iI_j(10)$ and ${}_iF_j(10)$ from table I, we can write the specialized formula for the almost perfect gas case

$$\frac{\delta^*_{AP}}{\sqrt{\mu_{so} g_{wo}/2\rho_{so}(du_e/dx)_{(x=o)}}}$$

$$= \frac{\delta^*_{AP}}{(1/\rho_{so})\sqrt{(\rho\mu)_{wo}/2(du_e/dx)_{(x=o)}}} = \frac{\delta^*_{AP}}{(1/\rho_{so})[(\rho\mu)_{so} g_{wo}/\sqrt{2}\, a_1^2 (2H_{st})]^{1/3}}$$

$$= 0.8045506 + \{[-0.062542936 - 0.01314843(\sigma_{wo} - 0.75)]g_1 + [0.9672229 - 0.57496903(\sigma_{wo} - 0.75)]\delta_1\}(hwo - 1) + \{[-0.29148217 - 0.09007333(\sigma_{wo} - 0.75)]m_1\delta_1 + [-0.027311875 + 0.03347037(\sigma_{wo} - 0.75)]g_1\delta_1 + 0.12066138 g_1^2 + [-0.07742036 - 0.04550108(\sigma_{wo} - 0.75)]\delta_1^2 + [-0.05836155 - 0.02286897(\sigma_{wo} - 0.75)]g_2\} \frac{(hwo - 1)^2}{2} + \{0.42222879 m_1^2\delta_1 + 0.01236733 m_1\delta_1^2 + 0.00946378 m_1 g_1\delta_1 + [-0.40057177 - 0.23221006(\sigma_{wo} - 0.75)]m_2\delta_1 - 0.14509049 g_1^3 + 0.03050273 g_1^2\delta_1 - 0.00904079 g_1\delta_1^2 + 0.04702020\delta_1^3 + [0.07293180 + 0.02999028(\sigma_{wo} - 0.75)]g_1 g_2 - 0.00701285 g_2\delta_1 + [-0.07886198 - 0.02546696(\sigma_{wo} - 0.75)]g_3\} \frac{(hwo - 1)^3}{6} + \{[-0.65220130 - 0.37522948(\sigma_{wo} - 0.75)]m_3\delta_1 + [0.88748589 + 0.52213220(\sigma_{wo} - 0.75)]m_1 m_2\delta_1 + [0.12060935 + 0.04324791(\sigma_{wo} - 0.75)]g_1 g_3\} \frac{(hwo - 1)^4}{24} + \{[1.68444624 + 0.98524535(\sigma_{wo} - 0.75)]m_1 m_3\delta_1 + [0.45202969 + 0.14630883(\sigma_{wo} - 0.75)]g_2 g_3\} \frac{(hwo - 1)^5}{120} + \{[6.67832064 + 3.91820519(\sigma_{wo} - 0.75)]m_2 m_3\delta_1 + [-1.32688310 - 0.45439610(\sigma_{wo} - 0.75)]g_1 g_2 g_3\} \frac{(hwo - 1)^6}{720} + \{[-31.28731960 - 18.55268985(\sigma_{wo} - 0.75)]m_1 m_2 m_3\delta_1\} \frac{(hwo - 1)^7}{5040} \quad (6.25)$$

Equation (6.25) is the working formula for the determination, for the almost perfect gas case, of δ^* in the vicinity of the stagnation point. For the perfect gas case, δ_p^* is determined by setting $\delta_1 = 1$ in equation (6.25). This equation has also been machine programmed. For hand computations the constants in the equation can be rounded off (see ch. 5, sect D). As with equation (6.12), combinations of

parameter values can make some terms in equation (6.25) either zero or negligibly small (for hand calculations). In figure 8(a) are shown some typical values of δ^* calculated from equation (6.25) (with a slightly different normalization).

2. Coefficient of Local Skin Friction

To obtain the coefficient of local skin friction in terms of our transformed variables, we begin by writing the shear equation

$$\bar{\tau} = \mu \frac{\partial u}{\partial y} \tag{6.26}$$

We transform to ξ,η coordinates using equation (2.40). We also make use of equation (2.21a) for $g(h; H_{st},P)$ and obtain

$$\bar{\tau} = \frac{(\rho\mu)_s g r u_e^2}{\xi^2 \sqrt{2}} f_{\eta\eta} \tag{6.27}$$

where, according to our nomenclature, $(\rho\mu)_s$ is $(\rho\mu)$ evaluated at H_{st} and at the pressure of the particular ξ station. At the wall we have

$$\bar{\tau}_w = \frac{(\rho\mu)_s g_w r u_e^2}{\xi^2 \sqrt{2}} f_{\eta\eta}(\xi,0) \tag{6.28}$$

We see that $\bar{\tau}$ and $\bar{\tau}_w$ are odd in ξ, having the value zero at the stagnation point (obvious from symmetry).

The coefficient of local skin friction is the wall shear normalized with a dynamic pressure. We will first normalize with the dynamic pressure, q_e, obtained from the velocity and density at the outer edge of the boundary layer.

$$q_e = \frac{1}{2} \rho_e u_e^2 \tag{6.29}$$

$$C_{fe} = \frac{\bar{\tau}_w}{q_e} \tag{6.30}$$

Substituting equations (6.28) and (6.29) in (6.30), we obtain

$$C_{fe} = \frac{\sqrt{2}\mu_s g_w r \delta(h_e)}{\xi^2} f_{\eta\eta}(\xi,0) \tag{6.31}$$

The coefficient, C_{fe}, becomes infinite at the stagnation point as $1/|\xi|$. (At this point the local Reynolds number based on x or ξ is zero.) The singularity can be removed by taking the product of C_{fe} and the square root of the local Reynolds number. The Reynolds number based on local values at the outer edge of the boundary layer is

$$Re = \frac{u_e \rho_e x}{\mu_e} \tag{6.32}$$

We see that Re is even in x or ξ. We form the dimensionless quantity

$$C_{fe}\sqrt{Re} = C_{fe}\sqrt{\frac{u_e\rho_e x}{\mu_e}} = \frac{\sqrt{2}\,\mu_s g_w r \delta(h_e)}{\xi^2}\sqrt{\frac{u_e\rho_e x}{\mu_e}}\; f_{\eta\eta}(\xi,0) \tag{6.33}$$

The product, $C_{fe}\sqrt{Re}$, as written is even in ξ. With some simplification we have

$$C_{fe}\sqrt{Re} = \frac{\sqrt{2}\, g_w r}{\xi^2}\sqrt{\frac{u_e(\rho\mu)_s x}{g(h_e)}}\; f_{\eta\eta}(\xi,0) \tag{6.34}$$

It is sometimes preferred to evaluate the coefficient of local skin friction in terms of a wall property dynamic pressure as follows:

$$q_w = \frac{1}{2}\rho_w u_e^2 \tag{6.35}$$

$$C_{fw} = \frac{\overline{\tau}_w}{q_w} \tag{6.36}$$

We substitute equations (6.28) and (6.35) in (6.36) and obtain

$$C_{fw} = \frac{\sqrt{2}\,\mu_w r}{\xi^2}\, f_{\eta\eta}(\xi,0) \tag{6.37}$$

Again, we will remove the singularity at the stagnation point by multiplying C_{fw} by a local Reynolds number, Rw, based on wall properties for this case.

$$Rw = \frac{u_e \rho_w x}{\mu_w} \tag{6.38}$$

Then we have

$$C_{fw}\sqrt{Rw} = C_{fw}\sqrt{\frac{u_e\rho_w x}{\mu_w}} = \frac{\sqrt{2}\,\mu_w r}{\xi^2}\sqrt{\frac{u_e\rho_w x}{\mu_w}}\, f_{\eta\eta}(\xi,0) \tag{6.39}$$

With a slight simplification we have

$$C_{fw}\sqrt{Rw} = \frac{\sqrt{2}\,r}{\xi^2}\sqrt{u_e(\rho\mu)_s g_w x}\, f_{\eta\eta}(\xi,0) \tag{6.40}$$

The product, $C_{fw}\sqrt{Rw}$, is, of course, even in ξ or x.

We will now specialize equations (6.34) and (6.40) for the vicinity of the stagnation point. We make use of equation (6.1), and we recall that near the stagnation point $r \approx x$, $u_e \approx (du_e/dx)_{(x=o)}x$, $g(h_e) = g(1) = 1$. We obtain:

$$C_{fe}\sqrt{Re} = 2\sqrt{2}\,\sqrt{g_{wo}}\,F''(0) \tag{6.41}$$

$$C_{fw}\sqrt{Rw} = 2\sqrt{2}\,F''(0) \tag{6.42}$$

Equations (6.41) and (6.42), valid in the region of the stagnation point, are the zero orders in ξ of the expansions of equations (6.34) and (6.40), respectively; the first neglected terms are of order ξ^2.

3. Heat-Transfer Coefficients

The rate of heat transfer per unit area by conduction in the boundary layer in the y direction, Q, is given by

$$Q = -k\frac{\partial T}{\partial y} \tag{6.43}$$

Using the relation

$$dH = c_p\,dT \tag{6.44}$$

we have

$$Q = -\frac{k}{c_p}\frac{\partial H}{\partial y} \tag{6.45}$$

We transform (6.45) to ξ,η coordinates using equation (2.40). We eliminate c_p and k by the relation $\sigma = c_p\mu/k$, and we insert $m(h; H_{st},P)$ as defined in equation (2.28), so that we have

$$Q = -\left(\frac{\rho\mu}{\sigma}\right)_s \frac{mr|u_e|H_{st}}{\xi^2\sqrt{2}} h_\eta \qquad (6.46)$$

where $(\rho\mu/\sigma)_s$ is evaluated at H_{st} and at the pressure of the ξ station. The value of Q is principally of interest at the wall since there it denotes the rate of heat transfer per unit area of wall from the wall into the boundary layer. For the rate of heat transfer at the wall, Q_w, we write

$$Q_w = -\left(\frac{\rho\mu}{\sigma}\right)_s \frac{m_w r|u_e|H_{st}}{\xi^2\sqrt{2}} h_\eta(\xi,0) \qquad (6.47)$$

In equations (6.46) and (6.47) we are maintaining an always positive value for r so we use $|u_e|$ to keep the sign of Q and Q_w consistent. As known from symmetry, Q and Q_w are even in ξ. A positive sign for Q_w means that heat is being transferred from the wall into the boundary layer and vice versa. As written, Q_w is a local value per unit area of wall.

A local Nusselt number can be written, although it may not be particularly useful at other than the stagnation point since the "history" of the boundary layer is involved in its evaluation. We can define a local Nusselt number based on wall properties as follows:

$$Nu_w = \frac{Q_w|x|c_{p_w}}{(H_w - H_{aw})k_w} \qquad (6.48)$$

where H_{aw} is the adiabatic wall enthalpy and is a function of ξ. This is the wall enthalpy distribution which will render the heat transfer, Q_w, zero at all ξ being considered. With a smooth distribution of wall enthalpies, $H_w(\xi)$, the ratio $Q_w/(H_w - H_{aw})$, will generally be positive, but wall enthalpy distributions are possible that will make this ratio, and therefore the Nusselt number, negative and even infinite. The absolute value, $|x|$, has been used to make Nu_w even in x and ξ.

We now substitute equation (6.47) in (6.48) and, after some simplification, we obtain:

$$Nu_w = \frac{\rho_s r u_e x}{\sqrt{2}\,\xi^2\delta_w(h_{aw} - hw)} h_\eta(\xi,0) \qquad (6.49)$$

where $h_{aw} = H_{aw}/H_{st}$ and ρ_s is evaluated at H_{st} and at the pressure of the ξ station. Nu_w will vanish at $\xi = 0$, but we can form an expression (which is even in ξ) that does not vanish as follows:

$$\frac{Nu_w}{\sqrt{Rw}} = \frac{Nu_w}{\sqrt{u_e \rho_w x/\mu_w}} \tag{6.50}$$

Using equation (6.49) we obtain, after some simplification

$$\frac{Nu_w}{\sqrt{Rw}} = \frac{r\sqrt{(\rho\mu)_s g_w u_e x}}{\sqrt{2}\,\xi^2(h_{aw} - hw)}\, h_\eta(\xi,0) \tag{6.51}$$

A local Stanton number based on wall properties can be defined as follows:

$$St_w = \frac{Q_w}{(H_w - H_{aw})\rho_w |u_e|} \tag{6.52}$$

(Using $|u_e|$ makes St_w even in ξ and x.) Using equation (6.48) we obtain the standard relationship

$$St_w = \frac{Nu_w}{Rw\,\sigma_w} \tag{6.53}$$

When we substitute equations (6.38) and (6.49) into (6.53) we obtain after some rearrangement

$$St_w = \frac{(\mu/\sigma)_s r m_w \delta_w}{\sqrt{2}\,\xi^2(h_{aw} - hw)}\, h_\eta(\xi,0) \tag{6.54}$$

where $(\mu/\sigma)_s$ is evaluated at H_{st} and at the pressure of the ξ station. As with the coefficient of local skin friction, the local Stanton number has an infinity as $1/|\xi|$ at the stagnation point. This can be removed by multiplying by the square root of the Reynolds number. From equation (6.53) we have

$$St_w \sqrt{Rw} = \frac{Nu_w}{\sqrt{Rw}\,\sigma_w} \tag{6.55}$$

This can be expressed in more detail by using equation (6.51).

The limitation on usefulness of the local Nusselt number also applies to the local Stanton number. Infinite or negative Stanton numbers are possible with certain distributions of wall enthalpy because of "historical effects." At the stagnation point, the quantity, $St_w \sqrt{Rw}$, can be considered as a useful coefficient.

We will now specialize equations (6.47), (6.51), and (6.55) for the vicinity of the stagnation point. We will use equation (6.1) and the stagnation point relations, $r \approx x$ and $u_e \approx (du_e/dx)_{(x=o)}x$. To obtain Q_w in several forms we will also use the relations given in equation (6.6). For Q_w near the stagnation point we have

$$Q_w = -\frac{\sqrt{2}\,(2H_{st})}{\sigma_{wo}}\left[\frac{1}{4}(\rho\mu)_{so}g_{wo}a_1\sqrt{2H_{st}}\right]^{2/3}G'(0)$$

$$= -\frac{H_{st}}{\sigma_{wo}}\sqrt{2(\rho\mu)_{wo}\left(\frac{du_e}{dx}\right)_{(x=o)}}\,G'(0) = -\frac{H_{st}}{\sigma_{wo}}\sqrt{2(\rho\mu)_{so}g_{wo}\left(\frac{du_e}{dx}\right)_{(x=o)}}\,G'(0) \tag{6.56}$$

Equation (6.56) is the zero-order term of the expansion in ξ of equation (6.47), the first neglected term being of order ξ^2. From the series expansion for G, equation (5.3), we know that when the wall enthalpy equals the stagnation enthalpy (hwo = 1), then $G(\eta) \equiv 1$ and $G'(\eta) \equiv 0$. Then, under these conditions, $Q_w = 0$ in equation (6.56). This means that at the stagnation point, the adiabatic wall enthalpy is equal to the stagnation enthalpy, or the dimensionless adiabatic wall enthalpy, h_{aw}, is unity.

Equations (6.51) and (6.55) specialize to

$$\frac{Nu_w}{\sqrt{Rw}} = \frac{\sqrt{2}}{(1 - hwo)}G'(0) \tag{6.57}$$

$$St_w\sqrt{Rw} = \frac{\sqrt{2}}{(1 - hwo)\sigma_{wo}}G'(0) \tag{6.58}$$

Equations (6.57) and (6.58) are valid in the vicinity of the stagnation point. They are the zero-order terms of the expansion in ξ of equations (6.51) and (6.55), respectively. The first neglected terms in (6.57), (6.58) are of order ξ^2. Equations (6.57) and (6.58) are not singular at hwo = 1, because, as seen from equation (5.5), the first term in $G'(0)$ is a term in (hwo - 1).

CHAPTER 7

ILLUSTRATIVE APPLICATIONS OF RESULTS

In this chapter we will demonstrate the use of the method we have developed. Two typical cases will be considered, one for a perfect gas and the other for a real gas. The method of procedure is essentially the same in both cases.

A. A TYPICAL PERFECT GAS

Our procedure will be the following. First the gas-property polynomials will be determined. Next the calculated dimensionless velocity and enthalpy profiles (in the transformed coordinate, η) will be compared with exact solutions. Finally we will transform back to the y coordinate system and will determine profiles of dimensionless velocity, enthalpy, density, a normalized transverse velocity, and the product of a normalized transverse velocity with density. We will also calculate skin-friction and heat-transfer coefficients and a normalized displacement thickness.

1. Determination of Gas-Property Polynomials

We will use flight in the earth's atmosphere for our example. Up to free-stream Mach numbers of about 5, we can consider that the actual air can be quite well approximated by a gas that is thermally and calorically perfect. Over the temperature range existing in the boundary layer in flight at a Mach number of 5, the compressibility function of air, $Z = P/\rho RT$, is essentially unity so that the perfect gas law holds: $P = \rho RT$ (ref. 10). Over this temperature range, the specific heat, c_p, does not change much, and we can consider it as having a constant average value. Then we can write

$$\frac{\rho_{so}}{\rho} = \delta(G) = G \tag{7.1}$$

which gives the polynomial coefficients (eq. (5.7c)):

$$\delta_1 = 1 \tag{7.2a}$$

$$\delta_2 = \delta_3 = 0 \tag{7.2b}$$

The viscosity can be well represented by Sutherland's law (ref. 10). This can be written (ref. 11) as:

$$\frac{\mu}{\mu_{so}} = \left(\frac{T}{T_{so}}\right)^{3/2} \left[\frac{1 + (198.6/T_{so})}{(T/T_{so}) + (198.6/T_{so})}\right] \tag{7.3}$$

Since we are considering the specific heat, c_p, to have a constant average value, we can write:

$$\frac{T}{T_{so}} = \frac{H}{H_{st}} = h = G \tag{7.4}$$

When we combine equations (7.1) and (7.4) with (7.3) we have:

$$\frac{\rho\mu}{(\rho\mu)_{so}} = g(G) = G^{1/2} \left[\frac{1 + (198.6/T_{so})}{G + (198.6/T_{so})}\right] \tag{7.5}$$

At this point we need to select a value for the stagnation temperature, T_{so}. We will use $T_{so} = 2520^{o}$ R which corresponds to flight at Mach 5, at an altitude of about 28,000 feet. (This altitude gives a stagnation pressure behind the normal shock, $P_{t2} = 10.6$ atmospheres, but this does not enter our calculations.) With these assumptions equation (7.5) yields

$$g(G) = \frac{\rho\mu}{(\rho\mu)_{so}} = G^{1/2} \left(\frac{1.0788}{G + 0.0788}\right) \tag{7.6}$$

We can see that a substantial variation in T_{so} will produce only a small change in equation (7.6).

Over the temperature range we are considering, the Prandtl number, σ, will change very little (ref. 10). For this case we will consider it to be constant. Then we have

$$m(G) = \frac{\rho\mu/\sigma}{(\rho\mu/\sigma)_{so}} = \frac{\rho\mu}{(\rho\mu)_{so}} = g(G) \tag{7.7}$$

So one polynomial will suffice to represent both $g(G)$ and $m(G)$.

We now want to put equation (7.6) in the form of the third-degree polynomial, equation (5.7b). In chapter 5, section E, we discussed several methods of obtaining the polynomial coefficients. In this case we will determine the coefficients by expanding equation (7.6) in a Taylor series about the point, $G = 1$. This will determine g_1 and g_2.

The coefficient, g_3, will be determined by forcing the polynomial to equal the exact value of g at $G = 0.1$. The coefficients we obtain are

$$g_1 = m_1 = -0.4269 \tag{7.8a}$$

$$g_2 = m_2 = 0.5415 \tag{7.8b}$$

$$g_3 = m_3 = -2.5047 \tag{7.8c}$$

Our polynomial is

$$g(G) = m(G) = 1 - 0.4269(G - 1) + \frac{0.5415}{2}(G - 1)^2 - \frac{2.5047}{6}(G - 1)^3 \tag{7.9}$$

In figure 6 we have plotted the Sutherland function, equation (7.6), and the polynomial (7.9). The comparison is considered quite satisfactory (particularly in view of the demonstration in appendix D, showing that final answers are not very sensitive to the shape of the gas property curves). We will make calculations for a number of different values of hwo, and, clearly, the best polynomial representation would

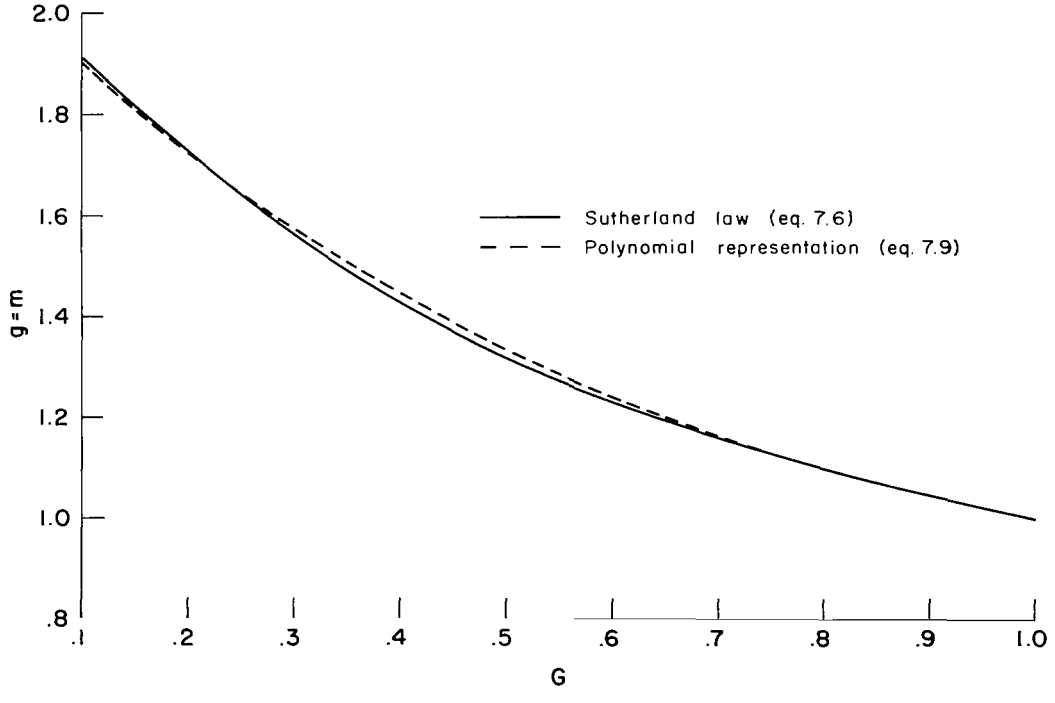


Figure 6.- Variation of $g = \rho\mu/(\rho\mu)_{so}$ and $m = (\rho\mu/\sigma)/(\rho\mu/\sigma)_{so}$ with G. Perfect gas: c_p, σ constant; $T_{so} = 2{,}520^{\circ}$ R.

be a slightly different polynomial for each hwo value. We will not seek this additional accuracy, but will use polynomial (7.9) for all values of hwo.

Since we have assumed a constant Prandtl number for this case, the wall Prandtl number, σ_{wo}, will be the Prandtl number through the boundary layer. We have made our calculations for two values of σ_{wo}, 0.70 and 0.75, as the comparison is of some interest. Most writers give the Prandtl number for low temperature air as approximately 0.70, although 0.75 is also found in the literature (ref. 10). It is believed that $\sigma_{wo} = 0.70$ is the more realistic value for air.

2. Comparison of Calculated Results With Exact Solutions

Our calculations of dimensionless enthalpy and velocity as functions of the transformed variable, η, will include cases with hwo values of 0.1, 0.2, 0.4, 0.6, 0.8, and 1.0. For the particular flight conditions we have assumed, some of the hwo are too low and some are too high to be considered practical. They have been included merely to illustrate the method and to show trends.

Our next step is to compare our calculated solutions with exact solutions. In obtaining exact solutions, we have solved equations (3.12), (3.13). (We recall that $f_o \equiv F$ and $h_o \equiv G$ in our nomenclature.) For $\delta(G)$ we have used equation (7.1), and for $m(G)$ and $g(G)$ we have used (7.6) and (7.7). So, for the exact solutions, the polynomial forms of the gas properties were not used; the original analytical forms for the gas properties were substituted directly into equations (3.12) and (3.13).

Calculated solutions were obtained from equations (5.3), (5.4), with values of G and F' determined at a number of points through the boundary layer. The comparisons are shown in figure 7 for both σ_{wo} values used. This comparison is considered to be very satisfactory. At the lowest values of hwo, there is a small discrepancy between exact and calculated values of F' in the outer portion of the boundary layer mainly because of the discarding of terms of lesser importance from the series solution (see fig. 1). The exact and calculated results match best near the wall, because the constants used in

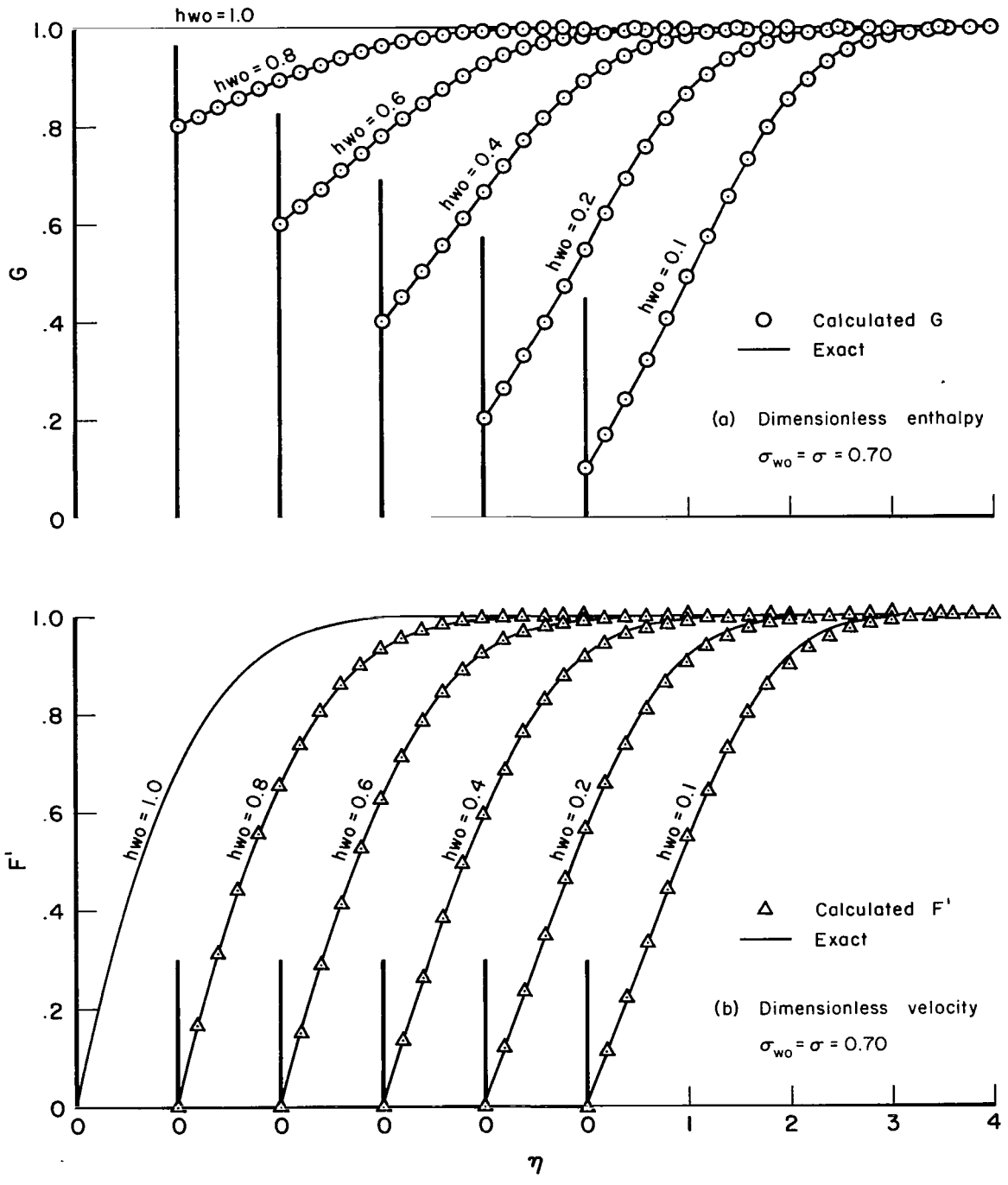


Figure 7.- Comparison of exact and calculated solutions. Perfect gas: $M_\infty = 5.0$; altitude = 28,000 ft; $T_{so} = 2{,}520^\circ$ R.

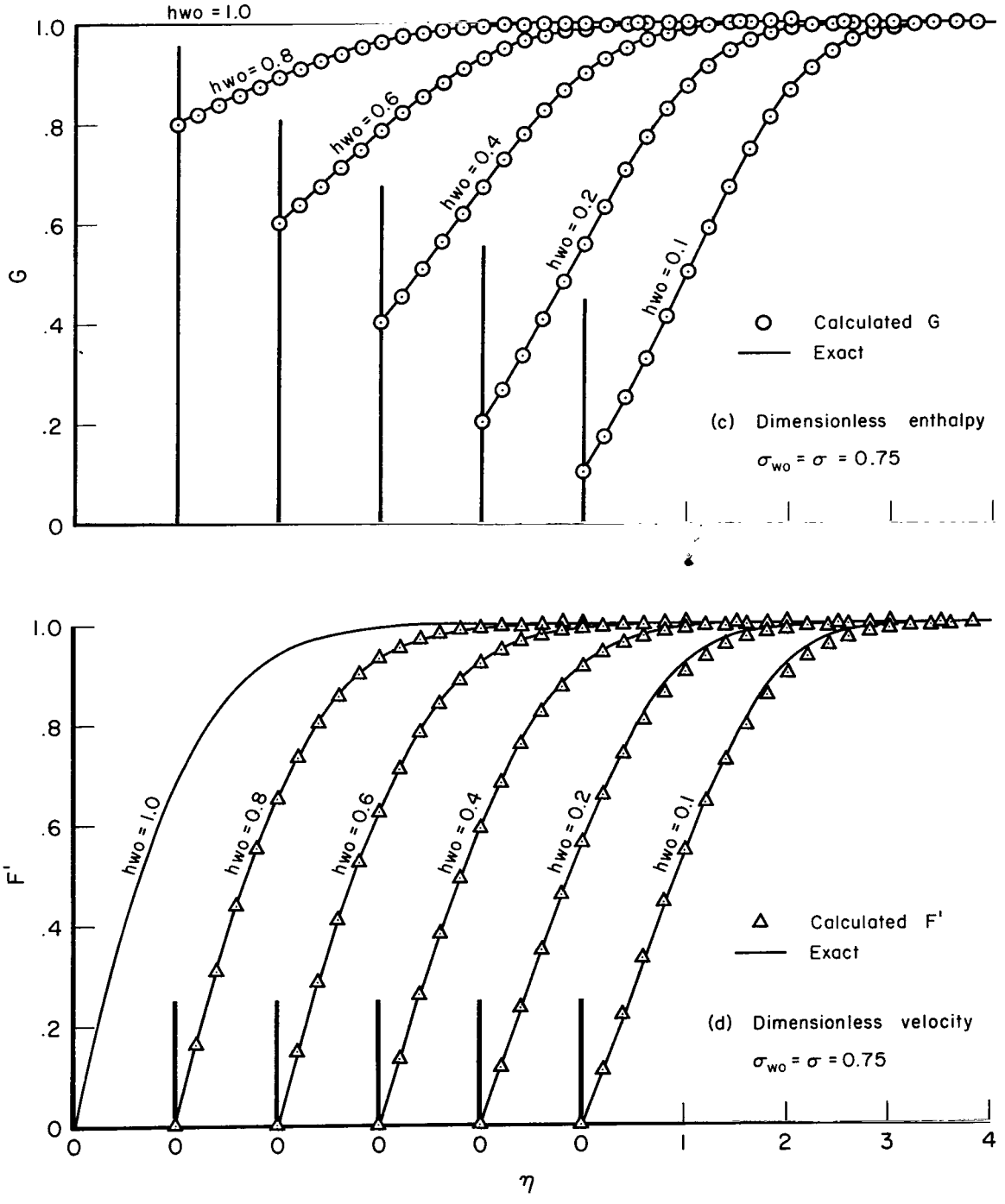


Figure 7.- Concluded.

truncating the series solutions were determined by matching calculated wall values with wall values from a number of exact solutions (tables 5-1 and 5-2). Curves for the case with hwo = 1 are shown in figure 7; at this hwo value, the exact and calculated solutions are identical. For a comparison of wall derivative values, G'(0) and F"(0), equations (5.5) and (5.6) were used for the calculated values. This comparison is shown in table 7-1. Again, the check is considered to be very satisfactory.

TABLE 7-1.- COMPARISON OF EXACT AND CALCULATED WALL DERIVATIVES FOR PERFECT GAS

M_∞ = 5.0; altitude = 28,000 ft; T_{SO} = 2,520° R

$\sigma = \sigma_{WO}$ (constant)	hwo	G'(0) exact	G'(0) calculated	Percent error in G'(0)	F"(0) exact	F"(0) calculated	Percent error in F"(0)
0.70	0.1	0.3247	0.3230	0.5	0.5089	0.5111	0.4
	.2	.3003	.3006	.1	.5592	.5647	1.0
	.4	.2441	.2436	.2	.6632	.6658	.4
	.6	.1729	.1727	.1	.7579	.7591	.2
	.8	.09056	.09057	0	.8454	.8458	0
	1.0	0	0	0	.9277	.9277	0
0.75	.1	.3336	.3323	.4	.5119	.5146	.5
	.2	.3084	.3091	.2	.5618	.5679	1.1
	.4	.2508	.2505	.1	.6652	.6684	.5
	.6	.1776	.1775	.1	.7593	.7610	.2
	.8	.09303	.09303	0	.8461	.8467	.1
	1.0	0	0	0	.9277	.9277	0

3. Calculated Coefficients and Profiles in Physical Coordinates

In figure 8 are plotted a number of calculated quantities as functions of a normalized y for σ_{WO} = 0.70. The corresponding curves for σ_{WO} = 0.75 are not shown as they are not discernibly different from those of figure 8. The calculated values were obtained from machine-programmed solutions using the permanent binary tape for the input of the universal functions.

Figure 8(a) shows the transformation from η to y. This was obtained from equation (6.12), but equation (6.8) could have been used instead. Both equations show that there are several ways of normalizing y into a dimensionless form. The form using a_1 was not used

because a_1 is a function of g_{wo} (eq. (6.5)), and g_{wo} depends on hwo. The normalization used for y in the figure is slightly different from a form shown in equation (6.12) in that $\sqrt{g_{wo}}$ has been moved to the right side of equation (6.12). This allows a more universal comparison for different hwo values when we assume a given flow and body shape with different hwo values applied. For a given flow, μ_{so} and ρ_{so} do not change with hwo, and for a given body shape, neglecting displacement-thickness interaction, the quantity $(du_e/dx)_{(x=o)}$ will be the same.

The curve, $y(\eta)$, for hwo = 1.0 is entirely linear, there being no compressibility effects (see eqs. (2.40), (6.12)). The other curves become linear at rather small η and normalized y values, showing that compressibility effects are virtually confined to the inner portion of the boundary layer (see figs. 7(a,b), 8(b,c)). On figure 8(a) are shown normalized values of the displacement thickness, δ^*, the normalization being the same as that used for y. The displacement thickness, δ^*, was obtained from equation (6.25) but could also be obtained from equation (6.23).

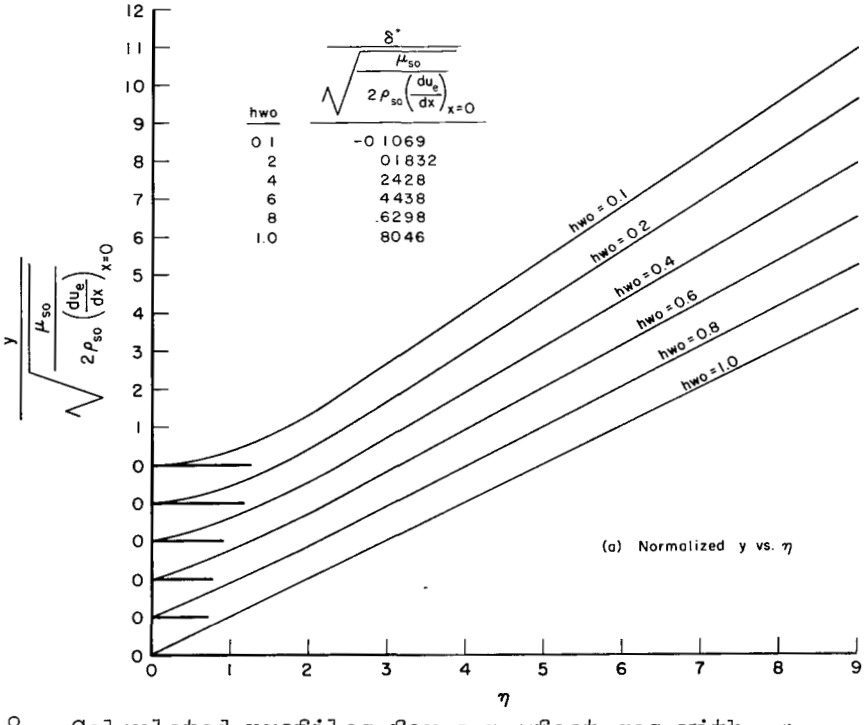


Figure 8.- Calculated profiles for a perfect gas with $\sigma_{wo} = \sigma = 0.70$: $M_\infty = 5.0$; altitude = 28,000 ft; $T_{so} = 2,520^\circ$ R.

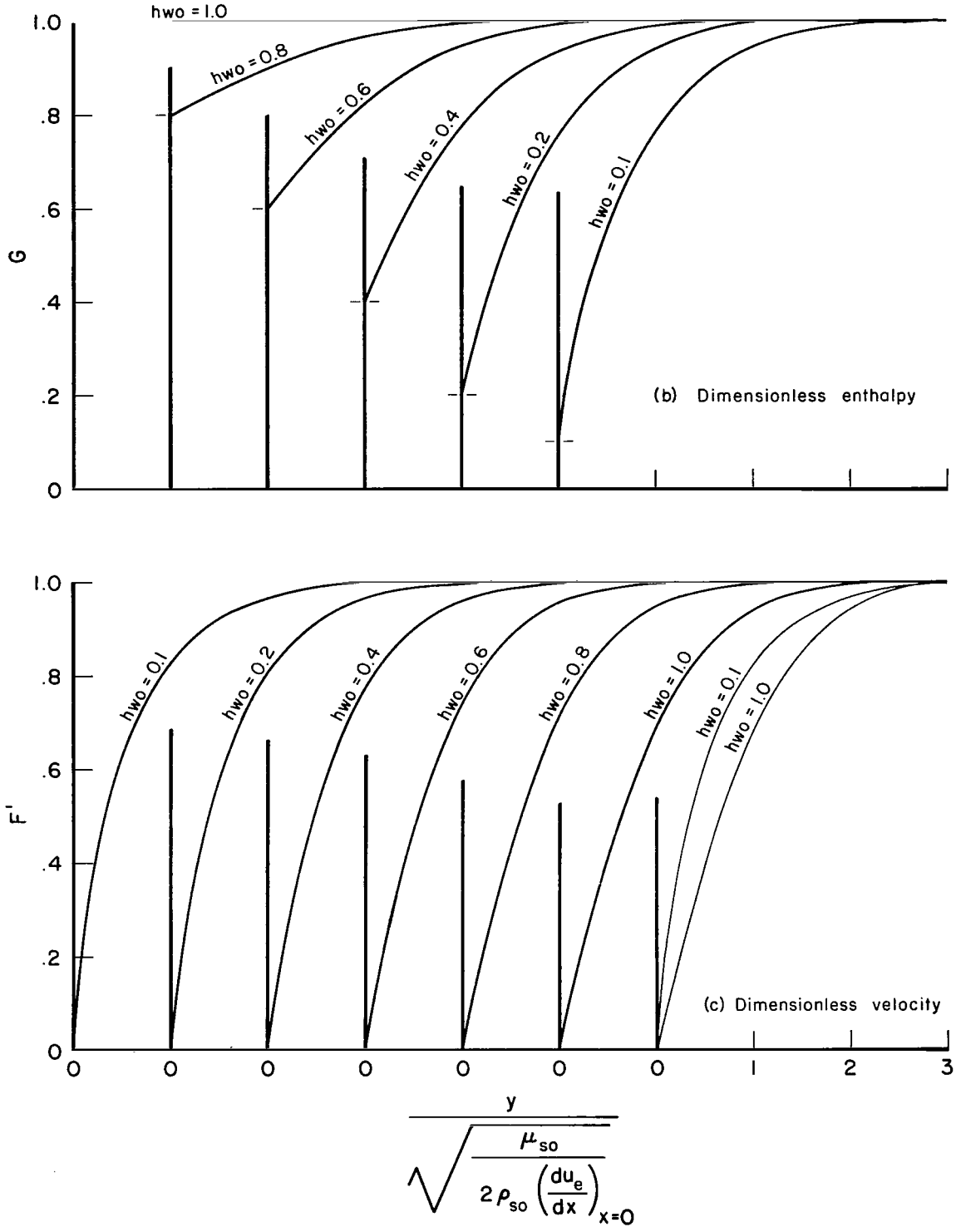


Figure 8.- Continued.

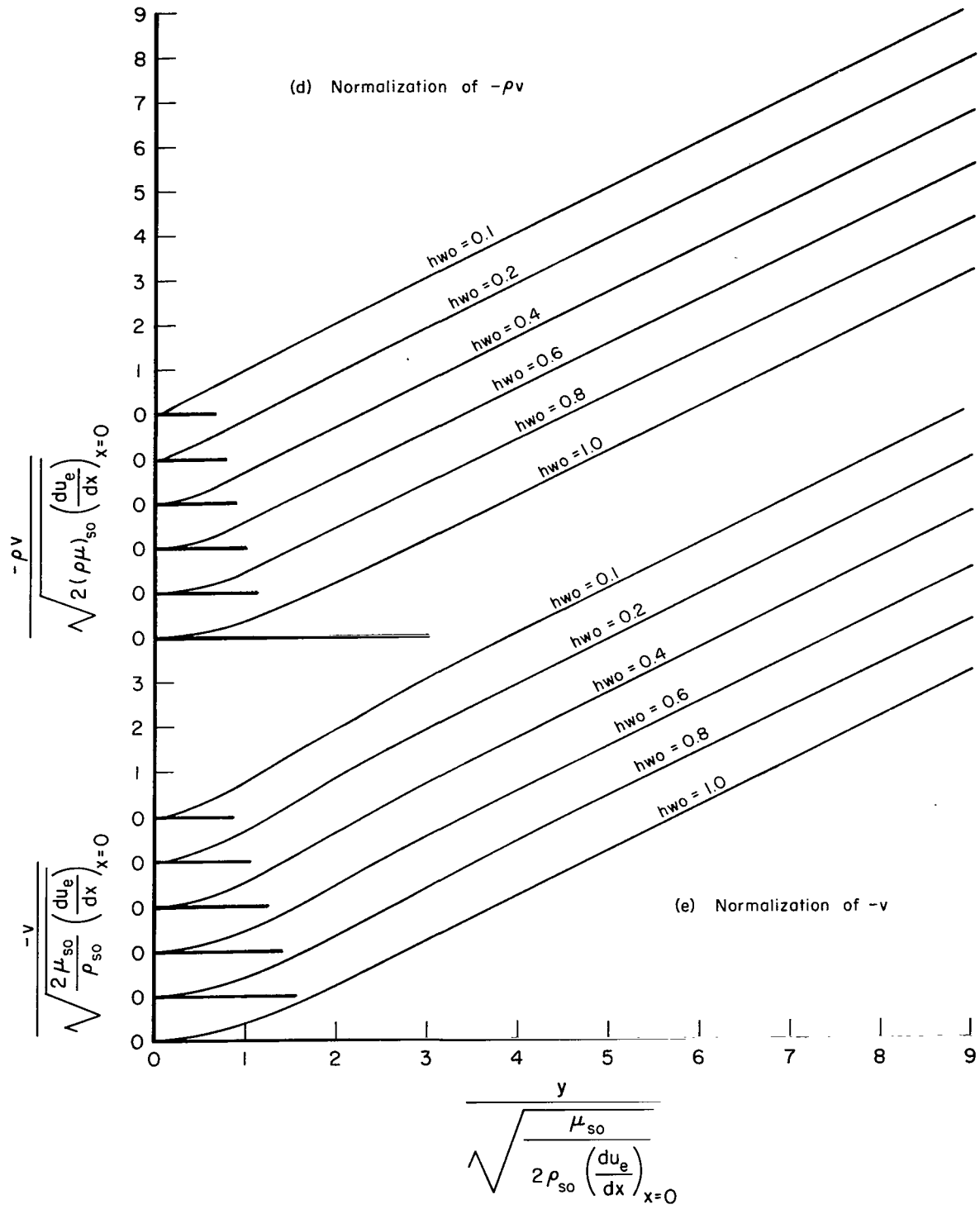


Figure 8.- Continued.

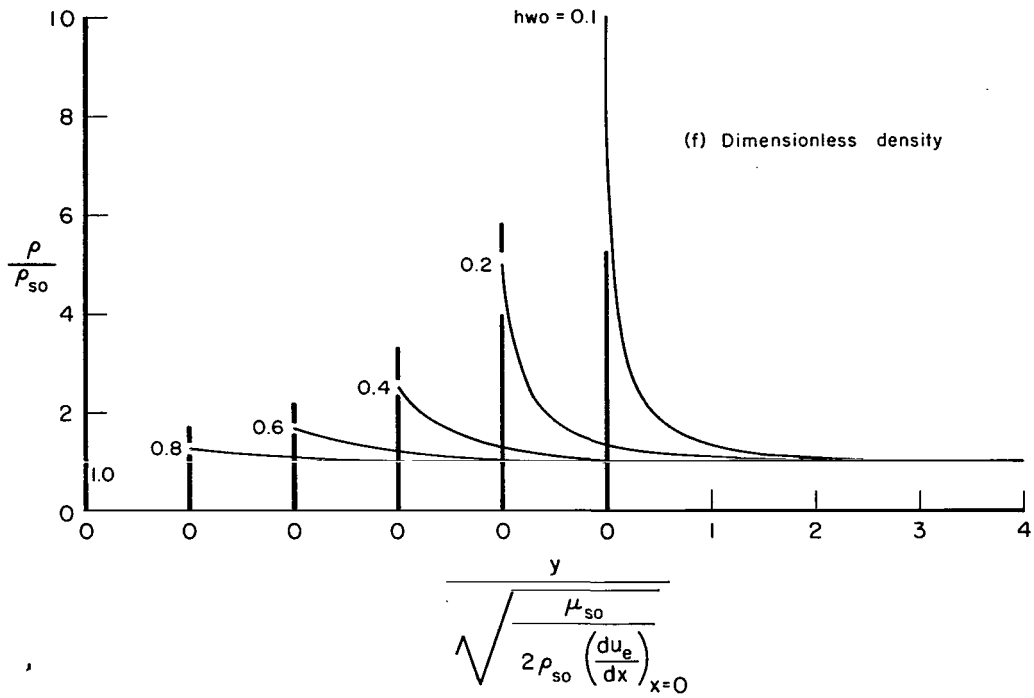


Figure 8.- Concluded.

The remaining parts of figure 8 are plotted on a normalized y basis (the physical coordinate) to show the trends of the data. In figures 8(b) and (c), respectively, the dimensionless enthalpy, G, and dimensionless velocity, F', are plotted against the normalized y.

In figure 8(d) we have plotted a normalized $-\rho v$ against the normalized y. The quantity, $-\rho v$, was obtained from equation (6.14). The case with the greatest compressibility effects, hwo = 0.1, has a curve that is essentially linear. All of the curves become linear at rather small values of the normalized y. In figure 8(e) we have a normalized form of the negative of the transverse velocity, -v, plotted against the normalized y. The transverse velocity was obtained from equation (6.18) using (6.19). These curves become essentially linear at small normalized y values, the case with hwo = 0.1 departing the least from linearity near the wall. In figure 8(f) we have plotted the dimensionless density, ρ/ρ_{SO}, against the normalized y. The density was obtained from equation (6.16) but could also be obtained from (6.15). This figure corresponds somewhat to figure 8(a) and shows that any major change in density occurs in the immediate vicinity of the wall.

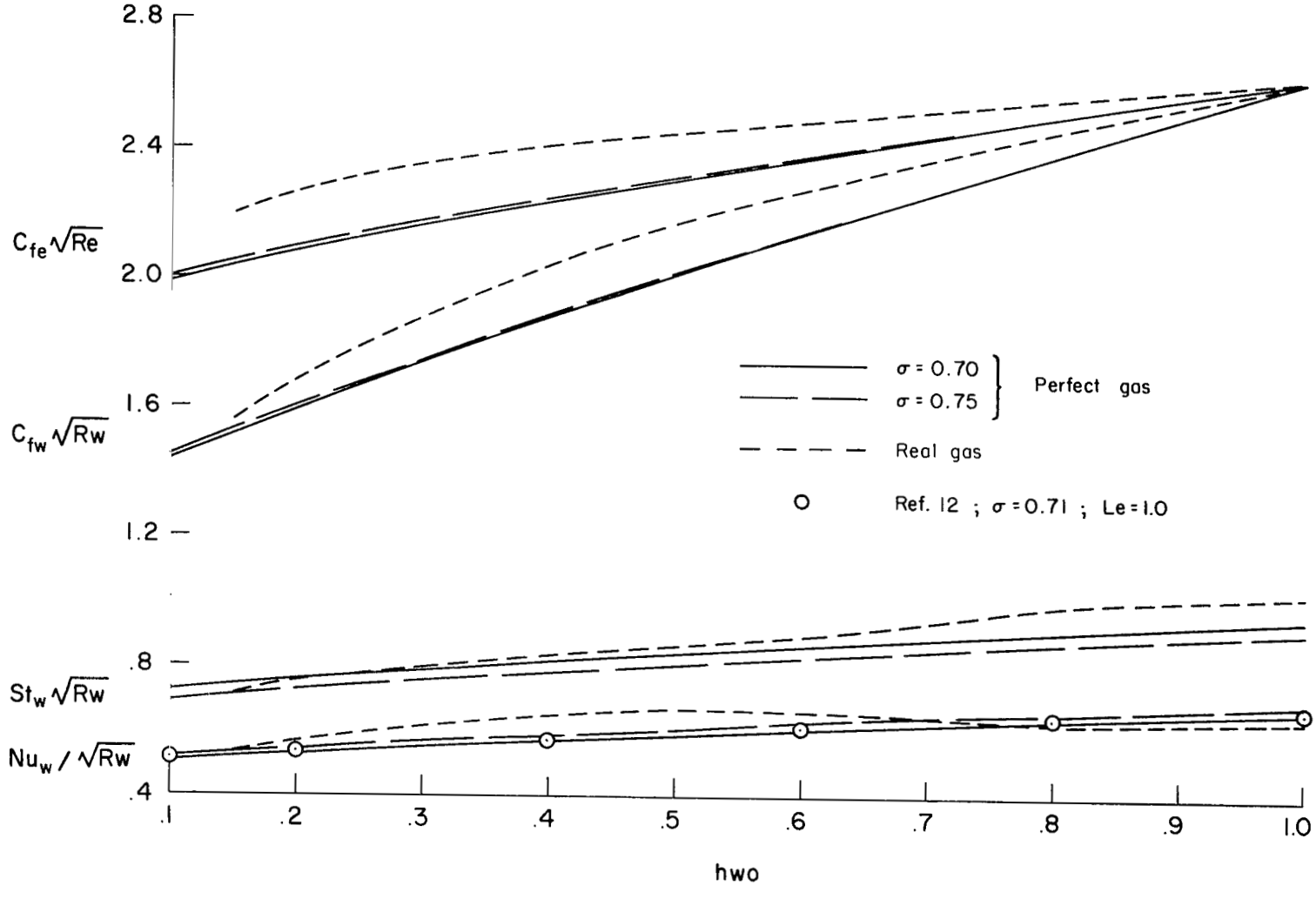


Figure 9.- Skin-friction and heat-transfer parameters vs. dimensionless wall enthalpy, hwo. Perfect gas: $M_\infty = 5.0$; altitude = 28,000 ft; $T_{so} = 2{,}520^\circ$ R. Real gas: $U_\infty = 9{,}750$ ft/sec; $M_\infty = 9.57$; altitude = 108,000 ft; $H_{st} = 2{,}000$ Btu/lb; $P_{t_2} = 1$ atm; $T_{so} = 5{,}740^\circ$ R.

In figure 9 for both σ_{wo} values, all the quantities $C_{fe}\sqrt{Re}$, $C_{fw}\sqrt{Rw}$, $Nu_w/\sqrt{Rw}$, and $St_w\sqrt{Rw}$ are plotted against hwo. The quantities plotted were obtained from equations (6.41), (6.42), (6.57), and (6.58), respectively. (Curves for a real gas discussed in section B, below, are also included in this figure.) As one might expect, the plots show that the value of the Prandtl number ($\sigma_{wo} = \sigma$ = constant for these perfect gas cases) has only a very small effect on the coefficient of local skin friction. The effect on heat transfer is small but noticeable. The $Nu_w/\sqrt{Rw}$ curves in figure 9 are compared with several values of this quantity obtained from reference 12.

By examining a number of numerical solutions, Fay and Riddell (ref. 12) developed several correlation formulas for stagnation-point heat transfer at the wall. Their numerical solutions used a constant Prandtl number, $\sigma = 0.71$, and several constant Lewis numbers. For the equilibrium boundary layer (and considered valid for the "frozen" boundary layer with a catalytic wall) with Lewis number, Le = 1, the correlation formula (eq. (58) of ref. 12) is, in our nomenclature,

$$\frac{Nu_w}{\sqrt{Rw}} = \frac{0.67}{(g_{wo})0.4}$$

Points calculated by the Fay and Riddell formula are shown in figure 9. It is seen that their values fall between ours for $\sigma = 0.70$ and 0.75, in extremely close agreement. It is of interest that the Fay and Riddell formula contains only one parameter, g_{wo} (which is equal to m_{wo} for a constant σ). This tends to confirm our conclusion in appendix D that between fairly wide limits the shapes of the gas property curves do not greatly affect the wall values of the heat transfer and the skin friction.

The curves plotted in figures 7, 8, and 9 are not universal because the flight conditions were specified. However, a moderate variation in flight conditions will produce only small changes in the final answers. The free-stream Mach number (or velocity of flight) and the altitude determine the stagnation temperature and enthalpy. The stagnation temperature determines the constant in the Sutherland viscosity law (7.3), and this will have some effect on the values of m_i and g_i,

but for moderate changes in T_{SO} this effect will not be severe. For small variations from the specified value of T_{SO}, the results shown in figures 7 through 9 can be considered good approximate answers. In discussing a small variation in T_{SO}, we are assuming that the gas is thermally and calorically perfect and that the Sutherland viscosity law still holds.

Our method of calculation seems best adapted to the type of perfect gas case we have been discussing. With this type, chemical effects are not important, and with third-degree polynomials, we can obtain good fits to the gas-property curves.

B. A TYPICAL REAL GAS

In this section we will carry through the calculations for a more severe case than the one we discussed in section A. Our polynomials will not give ideal fits to the gas-property curves, but a comparison with exact solutions will indicate that our calculated solutions furnish a reasonable approximation to the exact answers.

As our example, we will consider a vehicle flying in the earth's atmosphere with a velocity of 9,750 ft/sec at an altitude of approximately 108,000 ft. This corresponds to a free-stream Mach number, M_∞, of 9.57 (using the ARDC atmosphere). The stagnation enthalpy, H_{st}, is 2,000 Btu/lb and the stagnation pressure behind the normal shock wave, P_{t2}, is 1 atmosphere. At a stagnation enthalpy of 2,000 Btu/lb and a stagnation pressure of 1 atmosphere, the stagnation temperature is $5,740^\circ$ R (ref. 13).

1. Determination of Gas-Property Polynomials

We will use the charts of reference 13 for air to obtain the temperatures corresponding to our range of dimensionless enthalpies of interest ($G = h_o = H_o/2000$). In place of a formal equation of state, we obtain numerical values of the compressibility function, Z, (deviation from the perfect gas law) and thus obtain densities from reference 13. We then have a numerical tabulation of density as a function of the dimensionless enthalpy, G (not shown). We next obtain a numerical tabulation of $\delta(G) = \rho_{SO}/\rho$ against G (not shown). We will fit a polynomial to this.

Figure 10(a) shows a plot of $\delta(G)$ against G and the polynomial fit obtained. As can be seen, this matching is very good. The parameter values are:

$$\delta_1 = 0.6165 \qquad \delta_2 = -0.5536 \qquad \delta_3 = 0.6606$$

These values were obtained by forcing the polynomial to pass through two selected points (in addition to (1,1)), and by attempting to match the slope of the curve at (1,1). This polynomial will be used in calculating cases with various values of hwo.

For the gas property curve, $g(G) = \rho\mu/(\rho\mu)_{so}$, we already have numerical values of ρ/ρ_{so}, and we use reference 10 to obtain μ/μ_{so}. As given in reference 10, it turns out that the Sutherland formula can be used to give $\mu(T)$. From this we obtain numerical values of $\mu/\mu_{so}(G)$, since we already obtained a tabulated relationship between T and G from reference 13 (not shown). (In the perfect gas case we were able to obtain $\mu/\mu_{so}(G)$ directly, because we used a constant specific heat.) In figure 10(b) the function $g(G)$ is shown plotted. Also shown are plots of three different polynomials which will be used for different hwo cases. At the lower values of hwo it is not possible to get an ideal fit with a third-degree polynomial; a polynomial of higher degree would

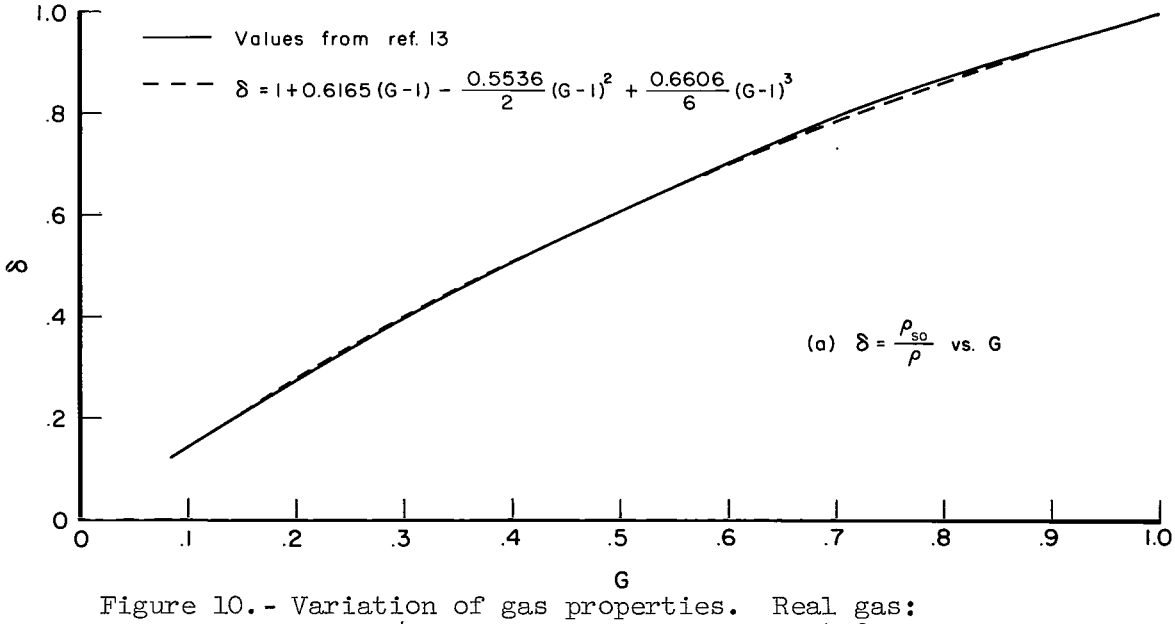


Figure 10.- Variation of gas properties. Real gas: H_{st} = 2,000 Btu/lb; Pt_2 = 1 atm; T_{so} = 5,740° R.

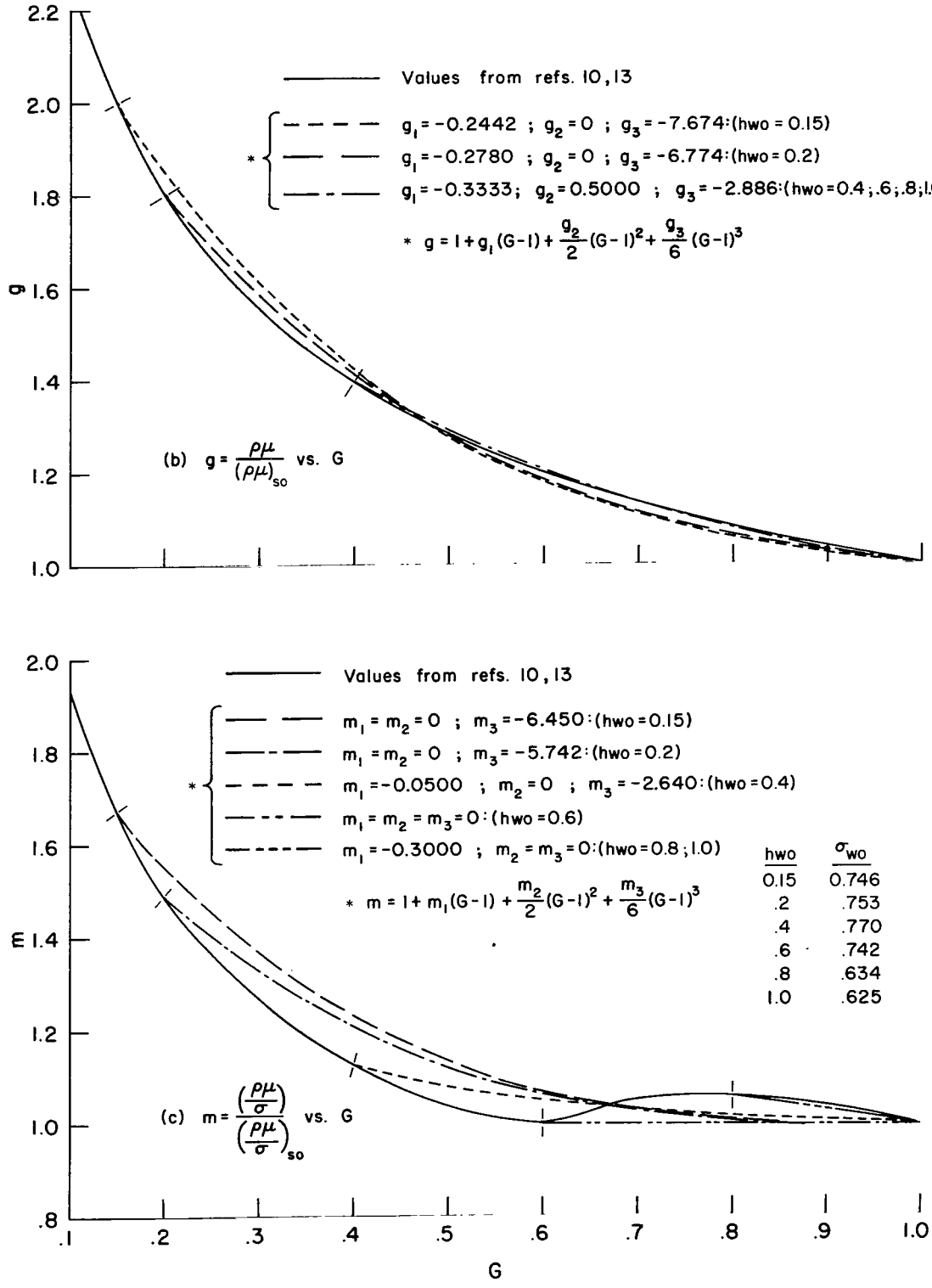


Figure 10.- Concluded.

be needed. Our polynomials will be approximations to the actual function, g(G), but we know from appendix D that final answers are not highly sensitive to the shapes of the gas property curves. For hwo = 0.15, which will be one of our cases, we have forced one polynomial to match the actual function at the points G = 0.15 and G = 0.5, and the third coefficient was determined by requiring that the shape of the polynomial reasonably approximate the g(G) curve. For another case with hwo = 0.2, we have done the same thing, this time requiring the polynomial to match the actual function at G = 0.2. For cases with hwo $\geq$ 0.4 we have obtained a good fit with a polynomial that we required to match the actual function at G = 0.4. The values of the coefficients in the polynomials are shown in figure 10(b).

In determining the gas property curve for m(G), we can use the relation

$$m(G) = \frac{\rho\mu/\sigma}{(\rho\mu/\sigma)_{so}} = \frac{\rho\mu}{(\rho\mu)_{so}} \frac{\sigma_{so}}{\sigma} = g \frac{\sigma_{so}}{\sigma}$$

Since we have evaluated g(G), we only need to determine σ_{so}/σ. We obtain numerical values of σ as a function of temperature from reference 10. We then tabulate σ_{so}/σ as a function of G (not shown), and obtain our tabulation of m(G) (not shown). This is plotted in figure 10(c). Before proceeding further it is desirable that we examine the meaning of the σ given in reference 10.

a. Use of an "effective" Prandtl number for the equilibrium boundary layer.- The flight case that we have chosen is in the regime where there is some oxygen dissociation (fig. 6 of ref. 10). We will assume that the air is in equilibrium throughout the boundary layer, so that the amount of dissociation is a function of the enthalpy (and pressure which has been taken as constant). In the colder portion of the boundary layer there will be recombination of the dissociated oxygen atoms with a consequent liberation of energy dissociation. For air in equilibrium, Hansen (ref. 10) has calculated a coefficient of thermal conductivity which is actually the sum of the usual conductivity coefficient and a conductivity coefficient that accounts for diffusion and the heat of recombination. Similarly, Hansen's specific heat, c_p, is a

specific heat that accounts for the variation of concentration of constituents with temperature. So Hansen's Prandtl number is actually an "effective" Prandtl number, and we can write

$$\sigma_{eff} = \frac{c_{p_{eff}} \mu}{k_{eff}} \tag{7.10}$$

where the subscript eff means that the quantity is evaluated as in reference 10. This is the Prandtl number that we have used in evaluating m(G) which we have plotted in figure 10(c). This determination of the Prandtl number allows us to use our energy equation as is, and we account for diffusion and recombination in the equilibrium boundary layer.

The method of lumping thermal conductivity coefficients was apparently first proposed by Hirschfelder (ref. 14) and modified by Butler and Brokaw (ref. 15). The method is clearly explained by Cohen (ref. 16). It should, of course, be noted that the Prandtl number and the coefficient of thermal conductivity have not been given their usual definitions by Hansen. Cohen (ref. 16) has termed Hansen's definitions "effective" values, which is probably good nomenclature. The usual definitions of Prandtl number, specific heat, and coefficient of thermal conductivity are called "partial" values by Hansen. In the lower temperature regimes with negligible dissociation, the "effective" coefficients are identical with the usual (or "partial" values). In the higher temperature regimes the "effective" Prandtl number has a greater variation with temperature than the usual ("partial") Prandtl number (figs. 10 and 11(a) of ref. 10). As brought out in reference 16, the "effective" coefficient of thermal conductivity, and thus the effective Prandtl number, contains the Lewis number, Le, but there is no need to determine this quantity. In this treatment there is no assumption as to constancy of the Lewis number; it can be expected to vary through the boundary layer.

The curve of m(G) plotted in figure 10(c) is a gas property curve that is difficult to fit with a low-degree polynomial. (The inflection in the curve is due to dissociation of oxygen.) We will approximate this curve by five different polynomials for the cases that we want to calculate. The polynomial curves and the values of the coefficients

used are shown on the figure. Each polynomial was forced to pass through the exact curve at the G value corresponding to the hwo value for which we will calculate a case. None of the polynomials give an ideal fit. At the lower hwo values, 0.15 and 0.20, a polynomial of higher than third degree would be needed for an improved fit. At the higher hwo values, 0.40, 0.60, and 0.80, an ideal fit cannot be achieved because we have imposed a limitation on the signs of the coefficients (see table E-1). The approximate polynomial fits should give satisfactory answers, as we know that final answers are not particularly sensitive to the polynomial shape (appendix D). Values of the "effective" wall Prandtl number, σ_{wo}, for the hwo cases we will calculate are shown tabulated in figure 10(c). Some of these values (from ref. 10) for the lower temperatures are slightly higher than values given in most of the literature.

2. Comparison of Calculated Results With Exact Solutions

As we did with the perfect gas, we will compare our calculated solutions for dimensionless enthalpy and velocity as functions of η with exact solutions over a range of hwo values. The higher hwo values are not practical cases, but they are included to show the comparisons and trends. In the perfect gas case we had analytical relations for the gas properties; in this real gas case we do not have analytical relations, but we will use the gas-property polynomials in forming the exact solutions. This comparison between calculated and exact solutions will thus not involve the quality of the representation of the gas properties by polynomials, since both the exact and calculated solutions will use the same polynomials. The equations used for the exact solutions are (4.3) and (4.4) with boundary conditions (4.5). The calculated solutions were obtained from equations (5.3) and (5.4).

The comparison between calculated and exact solutions is presented in figure 11. As with the perfect gas case, at the lower hwo values these is some discrepancy at the outer edge of the boundary layer, due to discarding of small terms from the series solution. On the whole, this comparison is considered very satisfactory. Calculated values of the wall derivatives, $G'(0)$ and $F''(0)$, were obtained from equations

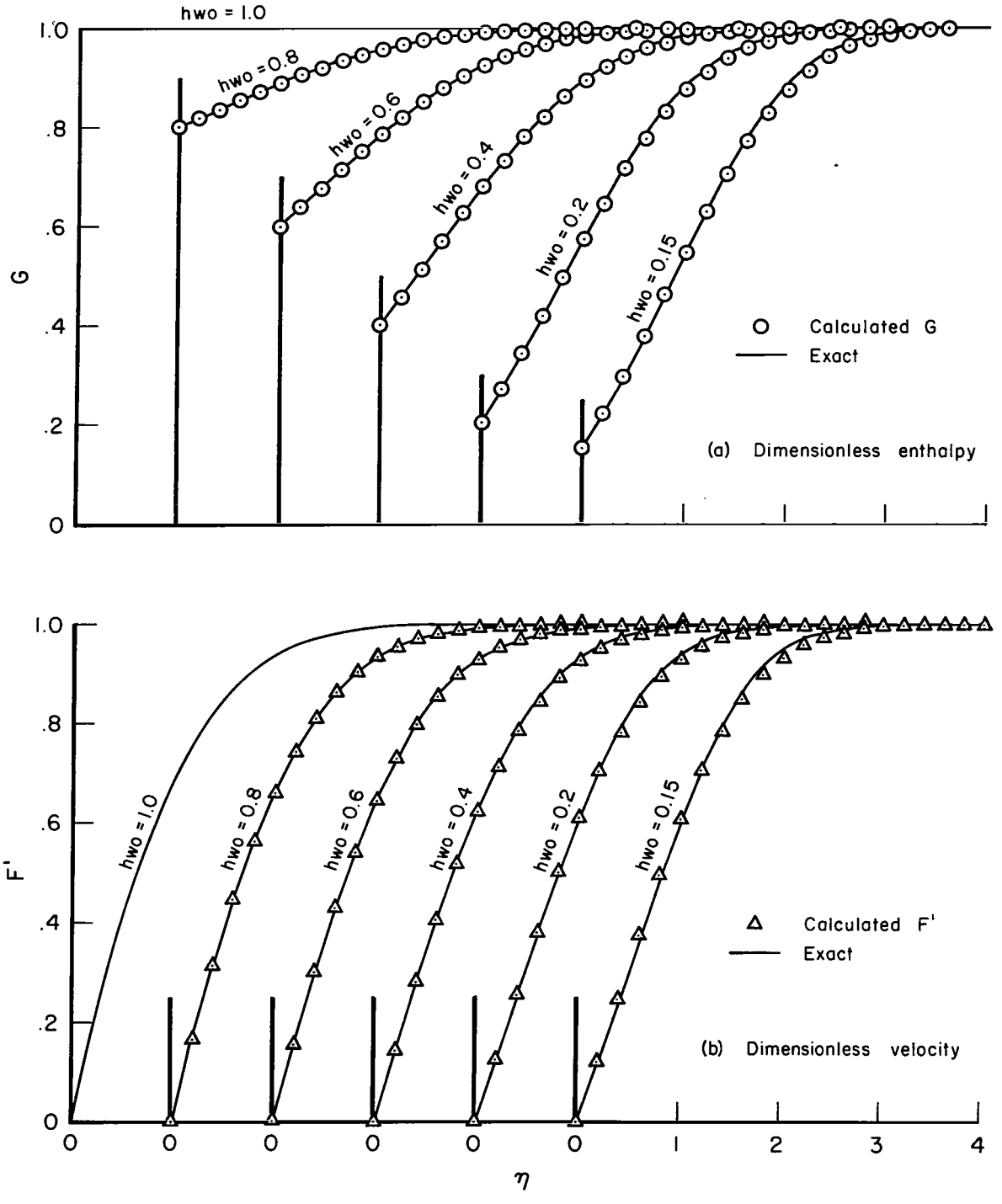


Figure 11.- Comparison of exact and calculated solutions. Real gas: U_∞ = 9,750 ft/sec; M_∞ = 9.57; altitude = 108,000 ft; H_{st} = 2,000 Btu/lb; P_{t_2} = 1 atm; T_{so} = 5,740° R.

(5.5) and (5.6). The calculated and the exact wall derivatives are compared in table 7-2. This comparison is also considered very satisfactory.

TABLE 7-2.- COMPARISON OF EXACT AND CALCULATED WALL DERIVATIVES FOR REAL GAS

U_∞ = 9,750 ft/sec; M_∞ = 9.57; altitude = 108,000 ft; H_{st} = 2,000 Btu/lb; P_{t2} = 1 atm; T_{so} = 5,740° R

hwo	σ	σ_{wo}, variable	G'(0) exact	G'(0) calculated	Percent error in G'(0)	F"(0) exact	F"(0) calculated	Percent error in F"(0)
0.15		0.746	0.3212	0.3216	0.1	0.5615	0.5521	1.7
.2		.753	.3166	.3215	1.5	.5983	.5979	.1
.4		.770	.2713	.2741	1.0	.7173	.7246	1.0
.6		.742	.1885	.1883	.1	.8015	.8056	.5
.8		.634	.08852	.08859	.1	.8706	.8711	.1
1.0		.625	0	0	0	.9277	.9277	0

3. Calculated Coefficients and Profiles in Physical Coordinates

In figure 12 are presented calculated results for the real gas in a manner that parallels figure 8. The normalization of the quantities is the same as used in the perfect gas case. All quantities were obtained from machine-programmed solutions using the permanent binary tape to input the universal functions. The normalized y was obtained from equation (6.8). The normalization of the quantity, $-\rho v$, was calculated from equation (6.14); the density ratio, ρ/ρ_{so}, was obtained from (6.15); the normalization of the negative transverse velocity, -v, was calculated from (6.18). The trends of the data shown in figure 12 are qualitatively similar to those of figure 8. Figures 12(a) and (f) show that compressibility effects are almost entirely confined to the portion of the boundary layer nearest the wall. For the case with hwo = 1.0, there are no compressibility effects.

For this real gas case, the quantities $C_{fe}\sqrt{Re}$, $C_{fw}\sqrt{Rw}$, $Nu_w/\sqrt{Rw}$, and $St_w\sqrt{Rw}$ have been plotted against hwo in figure 9 (dotted curves) along with the same quantities for the perfect gas discussed in section A, above. The plotted quantities were obtained, respectively, from equations (6.41), (6.42), (6.57), (6.58). The Nusselt number and

Stanton number curves for the real gas exhibit small inflections in the vicinity of the higher hwo values. As seen in the table in figure 10(c), the "effective" wall Prandtl number, σ_{wo}, dips sharply at hwo > 0.6. Also the curve, m(G), exhibits a reverse curvature. These factors can be expected to produce some effects on the Nusselt and Stanton numbers in the actual boundary layer. Our polynomial curves do not accurately follow the reverse curvature of m(G), but the coefficients, m_1, m_2, m_3, are affected by it. Our calculated solutions (and "exact" solutions in this case as well) utilize the polynomials and the σ_{wo} values tabulated in figure 10(c), so it is not surprising that slight inflections appear in the calculated results. It is not known how closely the inflections in the Nusselt and Stanton number curves would compare with results obtained from completely exact solutions which might be obtained by following the m(G) curve closely by numerical methods. This question is probably not important as the real gas cases with high hwo values cannot be considered as "practical" cases.

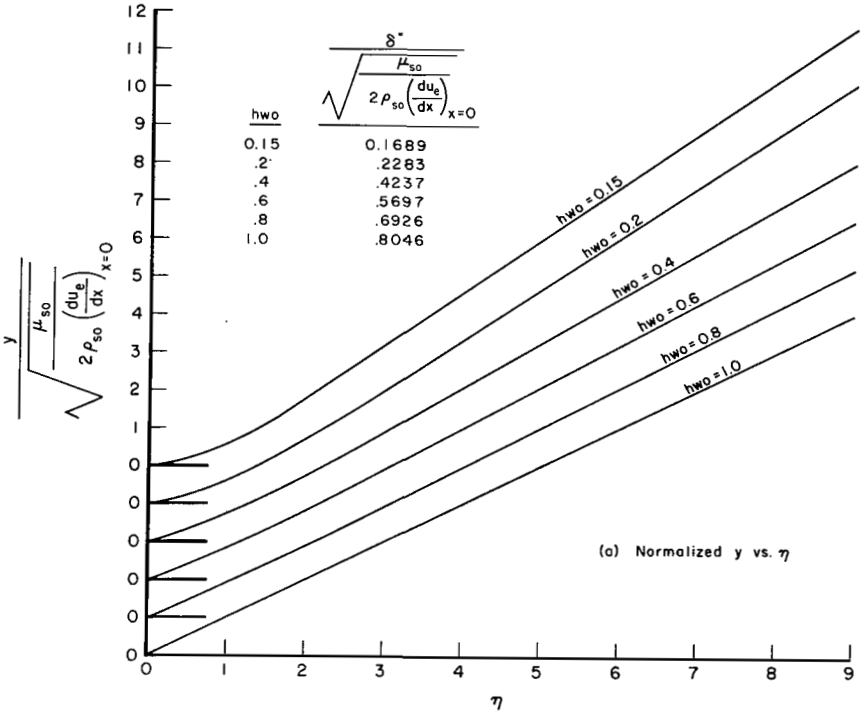


Figure 12.- Calculated profiles for real gas: U_∞ = 9,750 ft/sec; M_∞ = 9.57; altitude = 108,000 ft; H_{st} = 2,000 Btu/lb; P_{t_2} = 1 atm; T_{so} = 5,740° R.

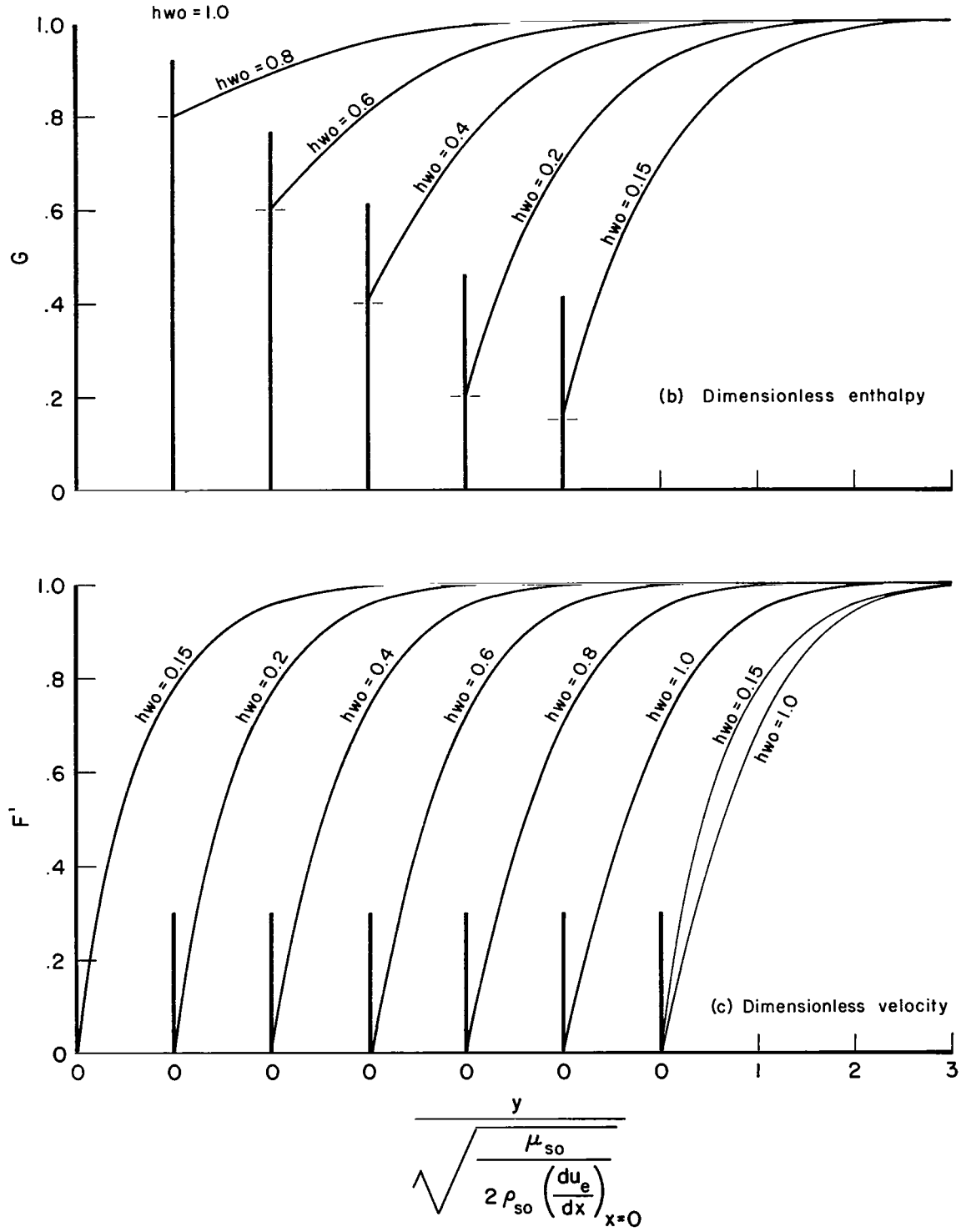


Figure 12.- Continued.

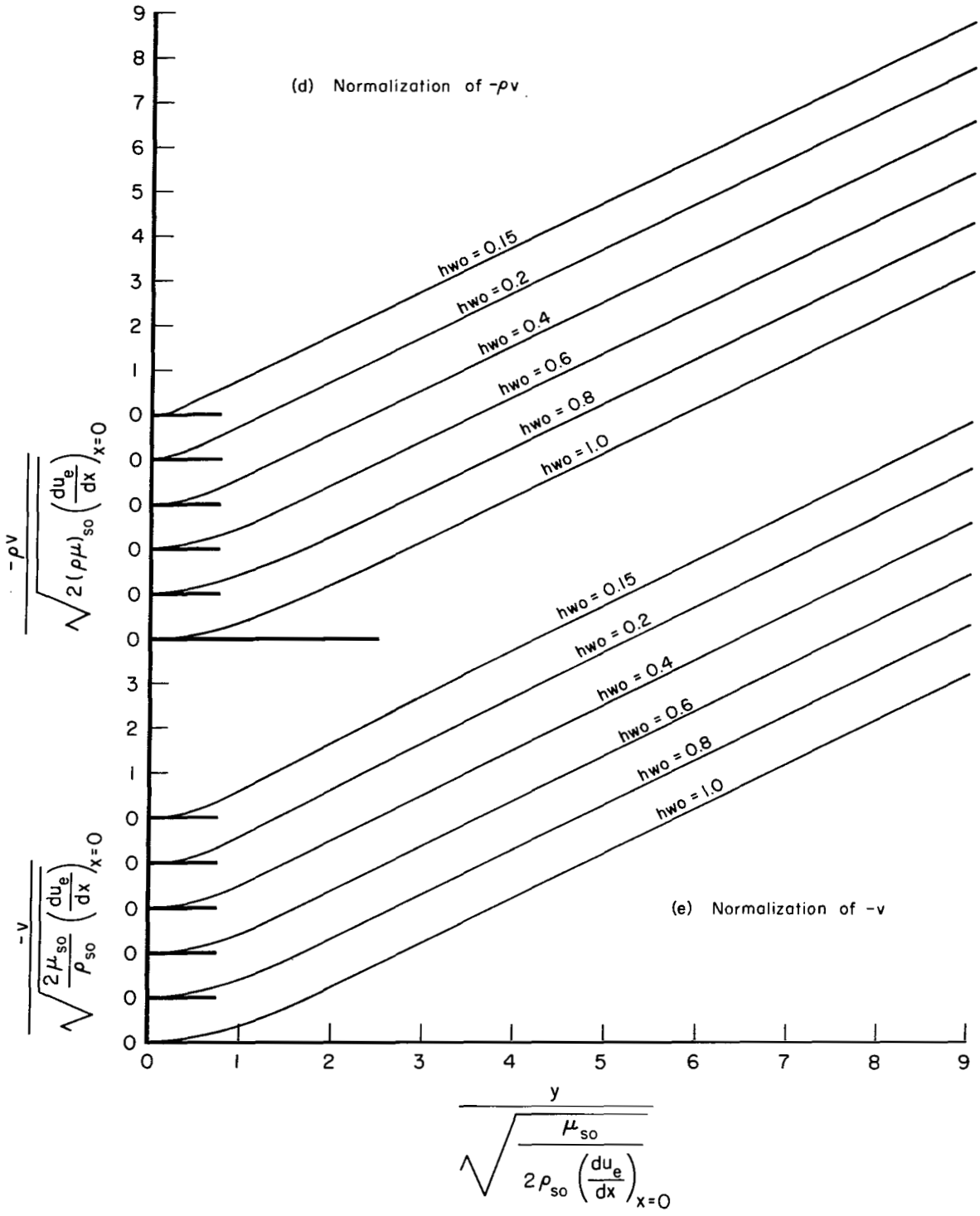


Figure 12.- Continued.

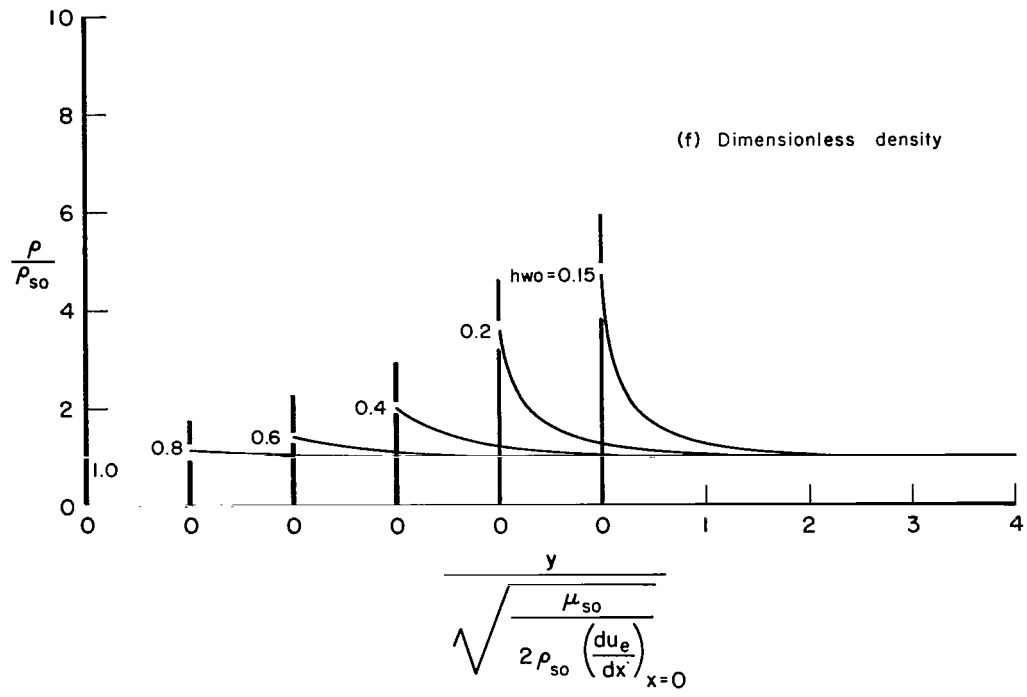


Figure 12.- Concluded.

Comparison of the results of the real gas with the perfect gas plotted in figure 9 indicates that a variation in the gas properties does somewhat affect both the skin friction and the heat-transfer coefficients. The variation shown ranges from zero to about 15 percent. At the lower hwo values the percentage variations between the two cases in the local skin-friction coefficients and the local heat-transfer coefficients are of about the same order of magnitude. (An examination of figures 6 and 10 shows that the m(G), g(G), and δ(G) curves are substantially different both in wall values and shape between the two cases.)

For the quantity $Nu_w/\sqrt{Rw}$, Fay and Riddell worked out a correlation formula for the equilibrium boundary layer to account for Lewis numbers, Le $\neq$ 1, but constant (eq. 60 of ref. 12). In our example case, taking the Lewis number, Le = 1.4 (an "average" value suggested by fig. 11(b) of ref. 10), this correction would increase the values of the Fay and Riddell points in figure 9 by about 3 percent. These points have not been plotted on figure 9. They would fall practically on top of our perfect gas curve for σ = 0.75, but are not too close to

our real gas curve for $Nu_w/\sqrt{Rw}$. Some of this difference may be due to our allowing σ to vary and not taking a constant Le in our calculations.

It should be pointed out again that our solution to this type of real gas case must be considered as approximate. We have seen that we cannot obtain ideal fits to all the gas-property curves with third-degree polynomials. Also our method, as set up, does not take into account finite chemical recombination rates, but is essentially restricted to the equilibrium boundary layer. As a further approximation, our calculations should furnish reasonable answers for wall heat-transfer values for a "frozen" boundary layer with a catalytic wall, but the profiles away from the wall will not be given with high accuracy (refs. 6, 12).

CHAPTER 8

DISCUSSION AND CONCLUSIONS

By a method of expansion in terms of a parameter, we have set up numerical solutions of a general form to the axially symmetric stagnation-point compressible laminar boundary-layer problem. The parameter of expansion that we have used is the deviation of the dimensionless wall enthalpy from unity, (hwo - 1). The gas properties needed have been represented as third-degree polynomials in powers of the deviation of the dimensionless enthalpy from unity, and the coefficients of the polynomials become parameters of the problem. The effects of all parameters are obtained by expanding in terms of the one parameter, (hwo - 1). The expansions have been kept to a reasonable length by truncations in which the highest-order term involving each parameter has been adjusted so that calculated answers agree well with the answers to exact solutions containing the various parameters.

The solutions have been obtained in terms of universal functions. These have been plotted (figs. 2 and 3), tabulated (table I), and stored on a permanent binary tape. The important working formulas to be used in obtaining solutions from the universal functions have been machine programmed; with machine calculations, the input to the formulas of the universal functions is obtained from a permanent binary tape. However, an automatic computing machine is not necessary for obtaining solutions; the values of the universal functions given in table I can be inserted in the formulas for hand computations. Some of the working formulas are quite lengthy; with machine calculations this makes little difference since these formulas have been programmed. In making hand calculations, one may have combinations of parameters that will render certain terms in the formulas very small or negligible, and one may judiciously drop these small terms depending on the accuracy desired.

Solutions can be calculated for any gas. For flight in the earth's atmosphere, the method, as set up, is best suited to cases with moderate free-stream Mach numbers, say around $M_\infty = 5$. For these cases, the gas-property functions can be closely fit with third-degree polynomials, and chemical effects are not especially important. At the higher Mach

numbers, the solutions should be considered to be reasonable approximations. The polynomial fits to the gas-property functions, in some of these cases, will not closely follow the shapes of the gas-property curves, but this does not have a large effect on the final answers. However, it is still recommended generally to try to obtain the best polynomial fits (see appendix D).

At the high Mach numbers, finite chemical recombination rates will be of increasing importance and the method does not take these finite rates into account. (This is not inherent in the method of solution, but is a consequence of the way in which the boundary-layer equations were originally written.) The calculations are for a gas in equilibrium when an "effective" Prandtl number is used which is obtained from a lumped coefficient of thermal conductivity that represents both conduction and diffusion (ref. 10). These equilibrium gas calculations will also give approximate values for heat transfer at the wall for a "frozen" gas with a catalytic wall (refs. 6 and 12).

The method can be extended to higher orders, allowing the use of gas-property polynomials of higher than third degree, but it may be doubted if this is worthwhile. Higher-order terms can be approximated as described in chapter 5, section F.

Numerical solutions have been obtained only for the stagnation-point problem. The boundary-layer equations can be expanded in powers of a transformed longitudinal variable to extend around a blunt body (indicated in appendix A). These equations can be solved in terms of universal functions by the method used for the stagnation-point equations, but the number of terms will be large.

Our method of calculation delivers the quantities of interest for the stagnation-point profile. As an important application, these quantities can be used to form starting profiles for finite-difference solutions extending around a blunt body. Approximate solutions (refs. 6 and 17) can also be started in this way.

For convenient reference a listing of the stagnation-region working formulas is given in table E-1 of appendix E. Also listed in table E-1 are the parameter limits which were used for the series truncations. The

selection of parameter values should normally be held within the limits indicated. An asterisk symbol in table E-1 indicates formulas which have been machine programmed, with the input of the universal functions from the permanent binary tape.*

*Available with computing program from Ames Research Center, NASA, Moffett Field, Calif.

APPENDIX A

SERIES EXPANSION OF THE MOMENTUM AND ENERGY EQUATIONS IN POWERS OF THE LONGITUDINAL VARIABLE

As noted in chapter 3, the momentum and energy equations (2.37) and (2.38) can be expanded in powers of the transformed longitudinal variable, ξ. This will produce pairs of coupled ordinary differential equations in the independent variable, η. From the symmetry of the problem we have observed that only even powers of ξ will give functions not identically zero.

A. GENERAL FORM OF EQUATIONS

We can substitute in equations (2.37) and (2.38) the expansions of all terms in powers of ξ, then separate out the different powers of ξ, and we have the equations for the various orders. This was done to obtain the zero-order equations (3.12) and (3.13) which are nonlinear.

To obtain the higher-order equations another procedure can be used. It can be recognized that expansions in powers of ξ are equivalent to Taylor's series in ξ about $\xi = 0$. So, we are able to take derivatives of the momentum and energy equations (2.37) and (2.38) with respect to ξ to various orders, then set $\xi = 0$, and obtain the pairs of coupled ordinary differential equations for the various orders. This is possibly more convenient than picking out powers of ξ.

The higher-order terms in the momentum and energy equations (in even powers of ξ only) deliver pairs of coupled, linear, ordinary differential equations. The left sides of the equations maintain the same general form. With increasing orders of ξ^i, the right sides of the equations become increasingly complicated as additional parameters affecting the solutions come into play. The right sides contain terms which are the solutions of lower orders, so all lower orders must be solved before solving a given order.

The general form of the equations other than zero order is as follows:

Momentum equations:

$$\left[\frac{g(h_o)}{g_{wo}}\right] f_i''' + \left[\left(\frac{\partial g}{\partial h}\right)_{h_o} \frac{h_o'}{g_{wo}} + f_o\right] f_i''$$

$$- \left[\left(1 + \frac{i}{2}\right) f_o'\right] f_i' + \left[\left(1 + \frac{i}{2}\right) f_o''\right] f_i$$

$$+ \left[\left(\frac{\partial g}{\partial h}\right)_{h_o} \frac{f_o''}{g_{wo}}\right] h_i'$$

$$+ \left[\left(\frac{\partial^2 g}{\partial h^2}\right)_{h_o} \frac{h_o' f_o''}{g_{wo}} + \left(\frac{\partial g}{\partial h}\right)_{h_o} \frac{f_o'''}{g_{wo}} + \frac{1}{2}\left(\frac{\partial \delta}{\partial h}\right)_{h_o}\right] h_i$$

$$= \left[\begin{array}{l}\text{Right side containing parameters} \\ \text{and terms from lower orders}\end{array}\right] \quad \text{(A1)}$$

Energy equations:

$$[m(h_o)]\, h_i'' + \left[2\left(\frac{\partial m}{\partial h}\right)_{h_o} h_o' + \sigma_{wo} m_{wo} f_o\right] h_i'$$

$$+ \left[\left(\frac{\partial^2 m}{\partial h^2}\right)_{h_o} (h_o')^2 + \left(\frac{\partial m}{\partial h}\right)_{h_o} h_o'' - \left(\frac{i}{2}\right) \sigma_{wo} m_{wo} f_o'\right] h_i$$

$$+ \left[\left(1 + \frac{i}{2}\right) \sigma_{wo} m_{wo} h_o'\right] f_i$$

$$= \left[\begin{array}{l}\text{Right side containing parameters} \\ \text{and terms from lower orders}\end{array}\right] \quad \text{(A2)}$$

The boundary conditions for equations (A1) and (A2) are as given in equation (3.11). These equations can be considered as "semi-universal." Their solutions depend ultimately on the solutions (for f_o and h_o) of the zero-order equations, (3.12) and (3.13), which are nonlinear. The values of f_o and h_o will depend on the dimensionless wall enthalpy as well as on the gas properties. It is seen in (A1) and

(A2) that f_o and h_o and their derivatives and also the gas properties appear in the coefficients on the left sides of these general form equations.

B. SECOND-ORDER EQUATIONS

As an illustration we can write the differential equations derived from the ξ^2 terms of the momentum and energy equations.

Momentum equation:

$$\left[\frac{g(h_o)}{g_{wo}}\right] f_2''' + \left[\left(\frac{\partial g}{\partial h}\right)_{h_o} \frac{h_o'}{g_{wo}} + f_o\right] f_2'' - [2f_o']f_2' + [2f_o'']f_2$$

$$+ \left[\left(\frac{\partial g}{\partial h}\right)_{h_o} \frac{f_o''}{g_{wo}}\right] h_2' + \left[\left(\frac{\partial^2 g}{\partial h^2}\right)_{h_o} \frac{h_o' f_o''}{g_{wo}} + \left(\frac{\partial g}{\partial h}\right)_{h_o} \frac{f_o'''}{g_{wo}} + \frac{1}{2}\left(\frac{\partial \delta}{\partial h}\right)_{h_o}\right] h_2$$

$$= h_{w2}\left(\frac{1}{g_{wo}^2}\left(\frac{\partial g}{\partial h}\right)_{hwo}\left\{\left(\frac{\partial g}{\partial h}\right)_{h_o} h_o' f_o'' + [g(h_o)]f_o'''\right\}\right)$$

$$+ a_1^2\left(-\delta_1[\delta(h_o)] + \rho_{so}(2H_{st})\left\{\frac{1}{g_{wo}}\left[\left(\frac{\partial^2 g}{\partial h \partial P}\right)_{h_o} h_o' f_o'' + \left(\frac{\partial g}{\partial P}\right)_{h_o} f_o'''\right]\right.\right.$$

$$\left.\left. + \frac{1}{2}\left(\frac{\partial \delta}{\partial P}\right)_{h_o} - \frac{1}{g_{wo}^2}\left(\frac{\partial g}{\partial P}\right)_{hwo}\left[\left(\frac{\partial g}{\partial h}\right)_{h_o} h_o' f_o'' + g(h_o) f_o'''\right]\right\}\right)$$

$$+ \frac{a_3}{a_1}\left\{\frac{1}{3}[(f_o')^2 - \delta(h_o)]\right\} + \sigma_{w2}[0] \qquad \text{(A3)}$$

Energy equation:

$$[m(h_o)]h_2'' + \left[2\left(\frac{\partial m}{\partial h}\right)_{h_o} h_o' + \sigma_{wo} m_{wo} f_o\right] h_2'$$

$$+ \left[\left(\frac{\partial^2 m}{\partial h^2}\right)_{h_o} (h_o')^2 + \left(\frac{\partial m}{\partial h}\right)_{h_o} h_o'' - \sigma_{wo} m_{wo} f_o'\right] h_2 + [2\sigma_{wo} m_{wo} h_o']f_2$$

$$= h_{w2}\left[-\sigma_{wo}\left(\frac{\partial m}{\partial h}\right)_{hwo} f_o h_o'\right] + a_1^2\left\{2\sigma_{wo} m_{wo}[\delta(h_o)]f_o' - \frac{4\sigma_{wo} m_{wo}[g(h_o)]}{g_{wo}}(f_o'')^2\right.$$

$$\left. + \rho_{so}(2H_{st})\left[\left(\frac{\partial^2 m}{\partial h \partial P}\right)_{h_o} (h_o')^2 + \left(\frac{\partial m}{\partial P}\right)_{h_o} h_o'' + \sigma_{wo}\left(\frac{\partial m}{\partial P}\right)_{hwo} f_o h_o'\right]\right\}$$

$$+ \frac{a_3}{a_1}[0] + \sigma_{w2}[m_{wo} f_o h_o'] \qquad \text{(A4)}$$

Because of their linearity, one can indicate the solutions to (A3) and (A4) in a split form.

$$h_2 = (hw2)_1 h_2 + (a_1^2)_2 h_2 + \left(\frac{a_3}{a_1}\right)_3 h_2 + (\sigma_{w2})_4 h_2 \qquad \text{(A5a)}$$

$$f_2 = (hw2)_1 f_2 + (a_1^2)_2 f_2 + \left(\frac{a_3}{a_1}\right)_3 f_2 + (\sigma_{w2})_4 f_2 \qquad \text{(A5b)}$$

The boundary conditions from (3.11) are as follows.
At the wall:

$$_i f_2(0) = {}_i f_2'(0) = 0 \qquad i: \ 1, 2, 3, 4 \qquad \text{(A6a,b)}$$

$$_1 h_2(0) = 1 \qquad {}_i h_2(0) = 0 \qquad i: \ 2, 3, 4 \qquad \text{(A6c,d)}$$

At the transition to the exterior flow:

$$_i f_2'(\infty) = 0 \qquad i: \ 1, 2, 3, 4 \qquad \text{(A6e)}$$

$$_2 h_2(\infty) = -2 \qquad {}_i h_2(\infty) = 0 \qquad i: \ 1, 3, 4 \qquad \text{(A6f,g)}$$

Boundary condition (A6f) comes from the right side of (3.11d), where, in this case $b_2 = 2a_1^2$ (see eq. (3.7)).

C. GAS PROPERTIES

The right sides of (A3) and (A4) contain the parameter σ_{w2} (eq. (3.8)). For most practical problems, the wall value of the Prandtl number σ_w would be so nearly constant that one would probably not need the σ_{w2} term.

The right sides of (A3) and (A4) also contain partial derivatives with respect to P of m, g, δ. These terms will usually be very small and, in most cases, can be dropped. Partial derivatives with respect to P affect the coefficients of the polynomials (3.1). We recall that m, g, δ are dimensionless ratios of gas properties at different enthalpies but at the same pressure. Over a considerable range, these ratios are not very sensitive to the pressure level. For example, with a gas that is thermally and calorically perfect, δ(h) = h with no pressure dependence. If a gas closely approximates a perfect gas, the Sutherland formula (containing no pressure term) can be used

to compute the viscosity (ref. 10); then $g(h)$ will not be pressure dependent. In this regime, the Prandtl number σ has little pressure dependence (ref. 10), so $m(h)$ will have very little pressure dependence. Concerning real gases, the writer checked plots for air of m, g, δ using data from references 10 and 13 and for carbon dioxide using unpublished data; and even into the ionization regime there was little pressure dependence. In figures 1 and 3 of reference 16 are plots that show this for air, for quantities that correspond to our δ and g. It is safe to say that for an approximate analysis, terms involving partial derivatives of m, g, δ with respect to P can generally be dropped.

D. NOTE ON CONVERGENCE

The expansions of f and h in powers of ξ, equations (3.2) and (3.3), can be considered to be valid regular or nonsingular perturbations around the axis, $\xi = 0$. The character of differential equations (2.37) and (2.38) is not changed with the substitution of the expansions in the equations and with the change from small ξ to $\xi = 0$. The momentum equation (2.37) is dominated by f terms and the highest derivative term is preserved as ξ approaches zero. The energy equation (2.38) is dominated by terms in h when ξ is small. When ξ becomes zero, the highest derivative term in h is preserved. Also no boundary conditions (2.41) are lost as ξ approaches zero.

The expansions for f and h, (3.2) and (3.3), are analogous to the Howarth-Blasius expansion for incompressible flow (ref. 1) and are quite similar to expansions in powers of a transformed longitudinal coordinate, τ (in our nomenclature) for compressible flow given in reference 7.

The expansions for f and h are valid only in their region of convergence, of course. Schlichting (ref. 1, pp. 132, 137) does not question the convergence of the Blasius series. It is apparent that the physical problem does not have singularities. However, it is possible that mathematical singularities may exist in the complex plane of ξ. This has been pointed out by Van Dyke (ref. 4) in connection with a meridian-length longitudinal coordinate and with a parabolic

longitudinal coordinate for incompressible flow over a parabolic blunt body (or body of revolution). Van Dyke has indicated that a singularity occurs somewhere on the circle, $|\chi_{ML}|/a = 0.62$, where χ_{ML} is the meridian-length longitudinal coordinate and a is the nose radius. When a parabolic longitudinal coordinate, χ_p, is used, a singularity appears to exist at $|\chi_p| = 1$ which corresponds to $|\chi_{ML}|/a = 1.15$. Van Dyke has shown that the circle of convergence can probably be enlarged to infinity by an Euler transformation to a new variable $\chi_p^2/(1 + \chi_p^2)$.

In the light of the above discussion, it is thought possible that our ξ may not be the best longitudinal coordinate from the standpoint of convergence. In the present work, a detailed analysis with numerical solutions is only being made for the stagnation-point (zero-order) equations. These are not affected by an Euler transformation. If solutions to the higher-order equations were to be obtained, consideration should be given to the possible advantage of the Euler transformation of the longitudinal variable.

On setting up series solutions to equations (2.37) and (2.38), it is necessary that any infinite series representation of the gas properties, such as would be indicated in equation (3.1) with infinite upper limits, be convergent (and the more rapidly the better). This possible problem is avoided by using finite polynomial representations of the gas properties as given in equation (3.1).

APPENDIX B

THE COMPONENTS OF THE STAGNATION-POINT EQUATIONS EXPANDED IN POWERS OF (hwo - 1)

We obtained solutions to the stagnation-point equations (4.3) and (4.4), with boundary conditions given by (4.5), by expanding the dimensionless enthalpy, G, and the dimensionless stream function, F, in powers of the parameter (hwo - 1). The expanded boundary conditions are given by equation (4.8) and the final forms of our series solutions are in equations (5.3) and (5.4).

The zero-order equations are (4.11) and (4.12) with boundary conditions (4.8a) and the left side of (4.8d). The higher-order equations are all of the form given by (4.13) and (4.14). These equations will be listed in this appendix. The equations, as written below, are complete through the third order. Some terms obtained from the equations below were dropped as negligible from the final series forms (5.3) and (5.4) (see also fig. 1). The nomenclature for terms dropped or approximated is explained in chapter 4, section E (e.g., aF_2). Above the third order, only the significant (non-negligible) terms have been retained on the right sides of the equations. Where a parenthesis on a right side contains no specific terms, the function corresponding to that parenthesis has been approximated or dropped. Approximations have been used for terms connected with the Prandtl number parameter, σ_1 (see ch. 5, sect. B).

For all orders higher than the first, the pertinent boundary conditions are homogeneous (4.8). These homogeneous boundary conditions apply to all components of the various orders, ${}_iF_j$, ${}_iG_j$, and will not be listed below.

A. FIRST ORDER, $(hwo - 1)^1$

Energy equation:

$$ {}_1G_1'' + [\sigma_o F_o]\,{}_1G_1' = 0 \qquad \text{(B1)} $$

Momentum equation:

$$F_1''' + [F_0]F_1'' - [F_0']F_1' + [F_0'']F_1$$
$$= (g_1)[F_0'''(1 - {}_1G_1) - F_0'' {}_1G_1'] + (\delta_1)[-(1/2){}_1G_1] \tag{B2}$$

$$F_1 = (g_1){}_1F_1 + (\delta_1){}_2F_1 \tag{B3}$$

Boundary conditions for the first-order equations are given by equations (4.8a), (4.8b), and the right sides of (4.8d,e).

B. SECOND ORDER, $(hwo - 1)^2$

Energy equation:

$$G_2'' + [\sigma_0 F_0]G_2' = (m_1)\{2[{}_1G_1''(1 - {}_1G_1) - ({}_1G_1')^2]\} + (g_1)(-2\sigma_0 {}_1G_1' {}_1F_1)$$
$$+ (\delta_1)(-2\sigma_0 {}_1G_1' {}_2F_1) + (2\sigma_1)(-F_0 {}_1G_1') \tag{B4}$$

$$G_2 = (m_1){}_1G_2 + (g_1){}_2G_2 + (\delta_1){}_3G_2 + (2\sigma_1){}_2G_1 \tag{B5}$$

($_2G_1$ is the one exception to our nomenclature system. It retains the second-order boundary condition, $_2G_1(0) = 0$ (see ch. 4, sect. E).)

Momentum equation:

$$F_2''' + [F_0]F_2'' - [F_0']F_2' + [F_0'']F_2$$
$$= (g_2)\{-2\,{}_1G_1\,{}_1G_1'\,F_0'' + F_0'''[1 - ({}_1G_1)^2]\} + (\delta_2)[-(1/2)({}_1G_1)^2]$$
$$+ (\delta_1{}^2)[-(1/2){}_3G_2 - 2\,{}_2F_1\,{}_2F_1'' + ({}_2F_1')^2] + (m_1g_1)(-F_0''\,{}_1G_2'$$
$$- F_0'''\,{}_1G_2) + (g_1{}^2)[-F_0''\,{}_2G_2' - F_0'''\,{}_2G_2 - 2\,{}_1G_1'\,{}_1F_1'' - 2\,{}_1G_1\,{}_1F_1'''$$
$$- 2F_0''\,{}_1F_1 - 2F_0\,{}_1F_1'' + 2F_0'\,{}_1F_1' - 2\,{}_1F_1\,{}_1F_1'' + ({}_1F_1')^2]$$
$$+ (g_1\delta_1)[-F_0''\,{}_3G_2' - F_0'''\,{}_3G_2 - 2\,{}_1G_1'\,{}_2F_1'' - 2\,{}_1G_1\,{}_2F_1''' - (1/2){}_2G_2$$
$$- 2\,{}_1F_1\,{}_2F_1'' - 2\,{}_2F_1\,{}_1F_1'' + 2\,{}_1F_1'\,{}_2F_1' + 2\,{}_2F_1'''\,]$$
$$+ (m_1\delta_1)[-(1/2){}_1G_2] + (g_1\sigma_1)[\quad\;] + (\delta_1\sigma_1)[\quad\;] \tag{B6}$$

$$F_2 = (g_1^2)_1F_2 + (g_1\delta_1)_2F_2 + (\delta_1^2)_3F_2 + (g_2)_4F_2 + (\delta_2)_5F_2$$
$$+ (g_1\sigma_1)_aF_2 + (\delta_1\sigma_1)_bF_2 + (m_1g_1)_cF_2 + (m_1\delta_1)_dF_2 \qquad \text{(B7)}$$

C. THIRD ORDER, $(hwo - 1)^3$

Energy equation:

$$G_3'' + [\sigma_oF_o]G_3' = (m_2)\Big(-3\{2\ {}_1G_1({}_1G_1')^2 + {}_1G_1''[({}_1G_1)^2 - 1]\}\Big)$$
$$+ (m_1^2)\{-3[{}_1G_2'(2\ {}_1G_1 + \sigma_oF_o) + {}_1G_1\ {}_1G_2'' + {}_1G_1''\ {}_1G_2]\}$$
$$+ (m_1g_1)\{-3[{}_2G_2'(2\ {}_1G_1' + \sigma_oF_o) + {}_1G_1\ {}_2G_2'' + {}_1G_1''\ {}_2G_2$$
$$+ 2\sigma_o\ {}_1G_1'\ {}_1F_1 + \sigma_o\ {}_1G_1'\ {}_cF_2 + \sigma_o\ {}_1F_1\ {}_1G_2']\}$$
$$+ (m_1\delta_1)\{-3[{}_3G_2'(2\ {}_1G_1' + \sigma_oF_o) + {}_1G_1\ {}_3G_2'' + {}_1G_1''\ {}_3G_2$$
$$+ 2\sigma_o\ {}_1G_1'\ {}_2F_1 + \sigma_o\ {}_1G_1'\ {}_dF_2 + \sigma_o\ {}_2F_1\ {}_1G_2']\}$$
$$+ (g_2)(-3\sigma_o\ {}_1G_1'\ {}_4F_2) + (\delta_2)(-3\sigma_o\ {}_1G_1'\ {}_5F_2)$$
$$+ (\delta_1^2)[-3\sigma_o({}_1G_1'\ {}_3F_2 + {}_2F_1\ {}_3G_2')]$$
$$+ (g_1^2)[-3\sigma_o({}_1G_1'\ {}_1F_2 + {}_1F_1\ {}_2G_2')]$$
$$+ (g_1\delta_1)[-3\sigma_o({}_1G_1'\ {}_2F_2 + {}_1F_1\ {}_3G_2' + {}_2F_1\ {}_2G_2')]$$
$$+ (m_1\sigma_1)[\qquad] + (g_1\sigma_1)[\qquad]$$
$$+ (\delta_1\sigma_1)[\qquad] + (\sigma_1^2)[\qquad] \qquad \text{(B8)}$$

$$G_3 = (m_1^2)_1G_3 + (m_1g_1)_2G_3 + (g_1^2)_3G_3 + (g_1\delta_1)_4G_3 + (\delta_1^2)_5G_3 + (m_1\delta_1)_6G_3$$
$$+ (m_2)_7G_3 + (\delta_2)_8G_3 + (g_2)_aG_3 + (m_1\sigma_1)_bG_3 + (g_1\sigma_1)_cG_3$$
$$+ (\delta_1\sigma_1)_dG_3 + (\sigma_1^2)_eG_3 \qquad \text{(B9)}$$

Momentum Equation:

$$
\begin{aligned}
&F_3''' + [F_0]F_3'' - [F_0']F_3' + [F_0'']F_3 \\
&= (g_3)\{-3({}_1G_1)^2\ {}_1G_1'\ F_0'' + F_0'''[1 - ({}_1G_1)^3]\} + (m_1g_2)[(-3\ {}_1G_1'\ F_0'' \\
&\quad - 3\ {}_1G_1\ F_0''')_1G_2 - 3\ {}_1G_1\ F_0''\ {}_1G_2'] + (g_1g_2)[(-3\ {}_1G_1'\ F_0'' \\
&\quad - 3\ {}_1G_1\ F_0''')_2G_2 - 3\ {}_1G_1\ F_0''\ {}_2G_2' + (-6\ {}_1G_1\ {}_1G_1' - 3F_0)_1F_1'' \\
&\quad - 3({}_1G_1)^2\ {}_1F_1''' - 3F_0''\ {}_1F_1 + 3F_0'\ {}_1F_1' - F_0''\ {}_aG_3' - F_0'''\ {}_aG_3 \\
&\quad + (-3\ {}_1G_1' - 3F_0)_4F_2'' - 3\ {}_1G_1\ {}_4F_2''' - 3F_0''\ {}_4F_2 + 3F_0'\ {}_4F_2' \\
&\quad - 3\ {}_1F_1''\ {}_4F_2 - 3\ {}_1F_1\ {}_4F_2'' + 3\ {}_1F_1'\ {}_4F_2'] + (g_2\delta_1)[(-3\ {}_1G_1'\ F_0'' \\
&\quad - 3\ {}_1G_1\ F_0''')_3G_2 - 3\ {}_1G_1\ F_0''\ {}_3G_2' + (-6\ {}_1G_1\ {}_1G_1' - 3F_0)_2F_1'' \\
&\quad - 3({}_1G_1)^2\ {}_2F_1''' - 3F_0''\ {}_2F_1 + 3F_0'\ {}_2F_1' - (3/2)_1G_1 - 3\ {}_2F_1''\ {}_4F_2 \\
&\quad - 3\ {}_2F_1\ {}_4F_2'' + 3\ {}_2F_1'\ {}_4F_2' - (1/2)_aG_3] + (m_2g_1)(-F_0''\ {}_7G_3' - F_0'''\ {}_7G_3) \\
&\quad + (m_1{}^2g_1)(-F_0''\ {}_1G_3' - F_0'''\ {}_1G_3) + (m_1g_1{}^2)[-F_0''\ {}_2G_3' - F_0'''\ {}_2G_3 \\
&\quad - 3\ {}_1F_1''\ {}_1G_2' - 3\ {}_1F_1'''\ {}_1G_2 + (-3\ {}_1G_1' - 3F_0)_cF_2'' - 3\ {}_1G_1\ {}_cF_2''' \\
&\quad - 3F_0''\ {}_cF_2 + 3F_0'\ {}_cF_2' - 3\ {}_1F_1''\ {}_cF_2 - 3\ {}_1F_1\ {}_cF_2'' + 3\ {}_1F_1'\ {}_cF_2'] \\
&\quad + (m_1g_1\delta_1)[-F_0''\ {}_6G_3' - F_0'''\ {}_6G_3 - 3\ {}_2F_1''\ {}_1G_2' - 3\ {}_2F_1'''\ {}_1G_2 \\
&\quad - (3/2)\ {}_1G_2 + (-3\ {}_1G_1' - 3F_0)_dF_2'' - 3\ {}_1G_1\ {}_dF_2''' \\
&\quad - 3F_0''\ {}_dF_2 + 3F_0'\ {}_dF_2' - 3\ {}_1F_1''\ {}_dF_2 - 3\ {}_2F_1''\ {}_cF_2 \\
&\quad - 3\ {}_1F_1\ {}_dF_2'' - 3\ {}_2F_1\ {}_cF_2'' + 3\ {}_1F_1'\ {}_dF_2' + 3\ {}_2F_1'\ {}_cF_2' - (1/2)\ {}_2G_3] \\
&\quad + (g_1{}^3)[-F_0''\ {}_3G_3' - F_0'''\ {}_3G_3 - 3\ {}_1F_1''\ {}_2G_2' - 3\ {}_1F_1'''\ {}_2G_2 + (-3\ {}_1G_1' \\
&\quad - 3F_0)_1F_2'' - 3\ {}_1G_1\ {}_1F_2''' - 3F_0''\ {}_1F_2 + 3F_0'\ {}_1F_2' - 6_1F_1\ {}_1F_1'' \\
&\quad + 3({}_1F_1')^2 - 3\ {}_1F_1''\ {}_1F_2 - 3\ {}_1F_1\ {}_1F_2'' + 3\ {}_1F_1'\ {}_1F_2'] \\
&\quad + (g_1{}^2\delta_1)[-F_0''\ {}_4G_3' - F_0'''\ {}_4G_3 - 3\ {}_1F_1''\ {}_3G_2' - 3\ {}_2F_1''\ {}_2G_2' \\
&\quad - 3\ {}_1F_1'''\ {}_3G_2 - 3\ {}_2F_1'''\ {}_2G_2 - (3/2)_2G_2 + (-3\ {}_1G_1' - 3F_0)_2F_2'' \\
&\quad - 3\ {}_1G_1\ {}_2F_2''' - 3F_0''\ {}_2F_2 + 3F_0'\ {}_2F_2' - 6_1F_1\ {}_2F_1'' - 6\ {}_2F_1\ {}_1F_1'' \\
&\quad + 6\ {}_1F_1'\ {}_2F_1' - 3\ {}_1F_1''\ {}_2F_2 - 3\ {}_2F_1''\ {}_1F_2 - 3\ {}_1F_1\ {}_2F_2'' - 3\ {}_2F_1\ {}_1F_2'' \\
&\quad + 3\ {}_1F_1'\ {}_2F_2' + 3\ {}_2F_1'\ {}_1F_2' - (1/2)_3G_3] + (g_1\delta_1{}^2)[-F_0''\ {}_5G_3' - F_0'''\ {}_5G_3 \\
&\quad - 3\ {}_2F_1''\ {}_3G_2' - 3\ {}_2F_1'''\ {}_3G_2 - (3/2)_3G_2 + (-3\ {}_1G_1' - 3Fo)_3F_2'' \\
&\quad - 3\ {}_1G_1\ {}_3F_2''' - 3F_0''\ {}_3F_2 + 3F_0'\ {}_3F_2' - 6\ {}_2F_1\ {}_2F_1'' + 3({}_2F_1')^2 \\
&\quad - 3\ {}_1F_1''\ {}_3F_2 - 3\ {}_2F_1''\ {}_2F_2 - 3\ {}_1F_1\ {}_3F_2'' - 3\ {}_2F_1\ {}_2F_2'' + 3\ {}_1F_1'\ {}_3F_2' \\
&\quad + 3\ {}_2F_1'\ {}_2F_2' - (1/2)_4G_3] + (g_1\delta_2)[-F_0''\ {}_8G_3' - F_0'''\ {}_8G_3 + (-3\ {}_1G_1' \\
&\quad - 3F_0)_5F_2'' - 3\ {}_1G_1\ {}_5F_2''' - 3F_0''\ {}_5F_2 + 3F_0'\ {}_5F_2' - (3/2)({}_1G_1)^2 \\
&\quad - 3\ {}_1F_1''\ {}_5F_2 - 3\ {}_1F_1\ {}_5F_2'' + 3\ {}_1F_1'\ {}_5F_2' - (3/2)_1G_1\ {}_2G_2] \\
&\quad + (\delta_1\delta_2)[-3\ {}_2F_1''\ {}_5F_2 - 3\ {}_2F_1\ {}_5F_2'' + 3\ {}_2F_1'\ {}_5F_2' - (3/2)_1G_1\ {}_3G_2 \\
&\quad - (1/2)_8G_3] + (\delta_1{}^3)[-3\ {}_2F_1''\ {}_3F_2 - 3\ {}_2F_1\ {}_3F_2'' + 3\ {}_2F_1'\ {}_3F_2' \\
&\quad - (1/2)_5G_3] + (m_1\delta_1{}^2)[-3\ {}_2F_1''\ {}_dF_2 - 3\ {}_2F_1\ {}_dF_2'' + 3\ {}_2F_1'\ {}_dF_2' \\
&\quad - (1/2)_8G_3] + \delta_3[-(1/2)({}_1G_1)^3] + (m_1\delta_2)[-(3/2)_1G_1\ {}_1G_2] \\
&\quad + (m_2\delta_1)[-(1/2)_7G_3] + m_1{}^2\delta_1[-(1/2)_1G_3] + (m_1g_1\sigma_1)[\quad] \\
&\quad + (m_1\delta_1\sigma_1)[\quad] + (g_1{}^2\sigma_1)[\quad] + (g_1\sigma_1{}^2)[\quad] + (g_1\delta_1\sigma_1)[\quad] \\
&\quad + (\delta_1{}^2\sigma_1)[\quad] + (\delta_1\sigma_1{}^2)[\quad] + (g_2\sigma_1)[\quad] + (\delta_2\sigma_1)[\quad]
\end{aligned}
\tag{B10}
$$

$$F_3 = (g_1^3)_1F_3 + (\delta_1^3)_2F_3 + (g_1^2\delta_1)_3F_3 + (g_1\delta_1^2)_4F_3 + (g_1g_2)_5F_3 + (g_1\delta_2)_6F_3 + (g_2\delta_1)_7F_3 + (\delta_1\delta_2)_8F_3 + (g_3)_9F_3 + (\delta_3)_{10}F_3 + (m_1g_2)_aF_3 + (m_2g_1)_bF_3 + (m_1^2g_1)_cF_3 + (m_1g_1^2)_dF_3 + (m_1g_1\delta_1)_eF_3 + (m_1\delta_1^2)_fF_3 + (m_1\delta_2)_gF_3 + (m_2\delta_1)_hF_3 + (m_1^2\delta_1)_iF_3 + (m_1g_1\sigma_1)_jF_3 + (m_1\delta_1\sigma_1)_kF_3 + (g_1^2\sigma_1)_lF_3 + (g_1\sigma_1^2)_mF_3 + (g_1\delta_1\sigma_1)_nF_3 + (\delta_1^2\sigma_1)_oF_3 + (\delta_1\sigma_1^2)_pF_3 + (g_2\sigma_1)_qF_3 + (\delta_2\sigma_1)_rF_3 \quad (B11)$$

D. FOURTH ORDER, $(hwo - 1)^4$

Starting with the fourth order, only significant terms are being retained on the right-hand sides, so the equations will be shorter (see fig. 1). Also the gas-property polynomials are terminated at the third degree, so terms with higher than third-order gas-property parameters will not appear.

Energy equation:

$$G_4'' + [\sigma_oF_o]G_4' = (m_3)[-4({}_1G_1)^3{}_1G_1'' - 12({}_1G_1)^2({}_1G_1')^2 - 4\sigma_o F_o {}_1G_1'] + (m_1m_2)[-4 {}_1G_1 {}_7G_3'' - 4 {}_1G_1'' {}_7G_3 - 8 {}_1G_1' {}_7G_3' - 6({}_1G_1)^2 {}_1G_2'' - 12 {}_1G_1 {}_1G_1'' {}_1G_2 - 24 {}_1G_1 {}_1G_1' {}_1G_2' - 12({}_1G_1')^2 {}_1G_2' - \sigma_o(4F_o {}_7G_3' + 6F_o {}_1G_2')] + (\delta_3)[-4\sigma_o {}_1G_1' {}_{10}F_3] + (m_2\sigma_1)[\quad] \quad (B12)$$

$$G_4 = (m_3)_1G_4 + (m_1m_2)_2G_4 + (\delta_3)_3G_4 + (m_2\sigma_1)_aG_4 \quad (B13)$$

Momentum equation:

$$F_4''' + [F_o]F_4'' - [F_o']F_4' + [F_o'']F_4 = (g_1g_3)[-4 {}_1F_1'' {}_9F_3 - 4 {}_1F_1 {}_9F_3'' + 4 {}_1F_1' {}_9F_3' - 4 {}_1G_1 {}_9F_3''' - 4 {}_1G_1' {}_9F_3'' - 4F_o'' {}_9F_3 - 4F_o {}_9F_3'' + 4F_o' {}_9F_3' - 4({}_1G_1)^3 {}_1F_1''' - 6F_o'''({}_1G_1)^2 {}_2G_2 - 6F_o''({}_1G_1)^2 {}_2G_2' - 12({}_1G_1)^2 {}_1G_1' {}_1F_1'' - 12 {}_1G_1 {}_1G_1' F_o'' {}_2G_2 - 4F_o'' {}_1F_1 - 4F_o {}_1F_1'' + 4F_o' {}_1F_1'] + (\delta_1\delta_3)[-4 {}_2F_1'' {}_{10}F_3 - 4 {}_2F_1 {}_{10}F_3'' + 4 {}_2F_1' {}_{10}F_3' - (1/2)_3G_4 - 3({}_1G_1)^2 {}_3G_2] + (g_3\sigma_1)[\quad] + (\delta_3\sigma_1)[\quad] + (g_1g_2\sigma_1)[\quad] + (\delta_1\delta_2\sigma_1)[\quad] \quad (B14)$$

$$F_4 = (g_1g_3)_1F_4 + (\delta_1\delta_3)_2F_4 + (g_3\sigma_1)_aF_4 + (\delta_3\sigma_1)_bF_4 + (g_1\delta_2\sigma_1)_cF_4 + (\delta_1\delta_2\sigma_1)_dF_4 \quad (B15)$$

E. FIFTH ORDER, $(hwo - 1)^5$

Energy equation:

$$G_5'' + [\sigma_o F_o]G_5'$$
$$= (m_1 m_3)[-5\ {}_1G_1\ {}_1G_4'' - 5\ {}_1G_1''\ {}_1G_4 - 10\ {}_1G_1'\ {}_1G_4' - 5\sigma_o F_o\ {}_1G_4'$$
$$- 10({}_1G_1)^3\ {}_1G_2'' - 30({}_1G_1)^2\ {}_1G_1''\ {}_1G_2 - 60({}_1G_1)^2\ {}_1G_1'\ {}_1G_2'$$
$$- 60\ {}_1G_1({}_1G_1')^2\ {}_1G_2 - 10\sigma_o F_o\ {}_1G_2'] + (m_1 m_2 \sigma_1)[\quad\]$$
$$+ (m_3 \sigma_1)[\quad\] \tag{B16}$$

$$G_5 = (m_1 m_3)\,{}_1G_5 + (m_1 m_2 \sigma_1)\,{}_aG_5 + (m_3 \sigma_1)\,{}_bG_5 \tag{B17}$$

Momentum equation:

$$F_5''' + [F_o]F_5'' - [F_o']F_5' + [F_o'']F_5$$
$$= (g_2 g_3)[-10\ {}_9F_3''\ {}_4F_2 - 10\ {}_4F_2''\ {}_9F_3 + 10\ {}_4F_2'\ {}_9F_3' - 10({}_1G_1)^2\ {}_9F_3'''$$
$$- 20\ {}_1G_1\ {}_1G_1'\ {}_9F_3'' - 10F_o''\ {}_9F_3 - 10F_o\ {}_9F_3'' + 10F_o'\ {}_9F_3'$$
$$- 10({}_1G_1)^3\ {}_4F_2''' - 30({}_1G_1)^2\ {}_1G_1'\ {}_4F_2'' - 10F_o''\ {}_4F_2 - 10F_o\ {}_4F_2''$$
$$+ 10F_o'\ {}_4F_2'] + (g_1 g_3 \sigma_1)[\quad\] + (\delta_1 \delta_3 \sigma_1)[\quad\] \tag{B18}$$

$$F_5 = (g_2 g_3)\,{}_1F_5 + (g_1 g_3 \sigma_1)\,{}_aF_5 + (\delta_1 \delta_3 \sigma_1)\,{}_bF_5 \tag{B19}$$

F. SIXTH ORDER, $(hwo - 1)^6$

Energy equation:

$$G_6'' + [\sigma_o F_o]G_6'$$
$$= (m_2 m_3)[-15({}_1G_1)^2\ {}_1G_4'' - 30\ {}_1G_1\ {}_1G_1''\ {}_1G_4 - 60\ {}_1G_1\ {}_1G_1'\ {}_1G_4'$$
$$- 30({}_1G_1')^2\ {}_1G_4 - 20({}_1G_1)^3\ {}_7G_3'' - 60({}_1G_1)^2\ {}_1G_1''\ {}_7G_3$$
$$- 120({}_1G_1)^2\ {}_1G_1'\ {}_7G_3' - 120\ {}_1G_1({}_1G_1')^2\ {}_7G_3 - \sigma_o(15F_o\ {}_1G_4'$$
$$+ 20F_o\ {}_7G_3')] + (m_1 m_3 \sigma_1)[\quad\] \tag{B20}$$

$$G_6 = (m_2 m_3)\,{}_1G_6 + (m_1 m_3 \sigma_1)\,{}_aG_6 \tag{B21}$$

Momentum equation:

$$
\begin{aligned}
&F_6''' + [F_0]F_6'' - [F_0']F_6' + [F_0'']F_6 \\
&\quad = (g_1g_2g_3)[-6\ {}_1G_1\ {}_1F_5''' - 6\ {}_1G_1'\ {}_1F_5'' - 60\ {}_9F_3''\ {}_4F_2 - 60\ {}_4F_2''\ {}_9F_3 \\
&\qquad - 6F_0''\ {}_1F_5 - 6F_0\ {}_1F_5'' + 60\ {}_4F_2'\ {}_9F_3' + 6F_0'\ {}_1F_5' - 15({}_1G_1)^2\ {}_1F_4''' \\
&\qquad - 60\ {}_1G_1\ {}_2G_2\ {}_9F_3''' - 60\ {}_1G_1\ {}_2G_2'\ {}_9F_3'' - 30\ {}_1G_1\ {}_1G_1'\ {}_1F_4'' \\
&\qquad - 60\ {}_2G_2\ {}_1G_1'\ {}_9F_3'' - 60\ {}_1F_1''\ {}_9F_3 - 15F_0''\ {}_1F_4 - 60\ {}_1F_1\ {}_9F_3'' \\
&\qquad - 15F_0\ {}_1F_4'' + 60\ {}_1F_1'\ {}_9F_3' + 15F_0'\ {}_1F_4' - 20({}_1G_1)^3\ {}_5F_3''' \\
&\qquad - 90({}_1G_1)^2\ {}_2G_2\ {}_4F_2''' - 60({}_1G_1)^2\ {}_1G_1'\ {}_5F_3'' - 90({}_1G_1)^2\ {}_2G_2'\ {}_4F_2'' \\
&\qquad - 180\ {}_1G_1\ {}_2G_2\ {}_1G_1'\ {}_4F_2'' - 60\ {}_1F_1''\ {}_4F_2 - 20F_0''\ {}_5F_3 - 60\ {}_1F_1\ {}_4F_2'' \\
&\qquad - 20F_0\ {}_5F_3'' + 60\ {}_1F_1'\ {}_4F_2' + 20F_0'\ {}_5F_3' - 15\ {}_1F_4''\ {}_4F_2 \\
&\qquad - 20\ {}_5F_3''\ {}_9F_3 - 20\ {}_9F_3''\ {}_5F_3 - 15\ {}_4F_2''\ {}_1F_4 - 6\ {}_1F_1''\ {}_1F_5 \\
&\qquad - 6\ {}_1F_1\ {}_1F_5'' + 20\ {}_5F_3'\ {}_9F_3' + 15\ {}_4F_2'\ {}_1F_4' + 6\ {}_1F_1'\ {}_1F_5'] \\
&\quad + (g_2g_3\sigma_1)[\qquad]
\end{aligned}
\tag{B22}
$$

$$
F_6 = (g_1g_2g_3)_1F_6 + (g_2g_3\sigma_1)_aF_6 \tag{B23}
$$

G. SEVENTH ORDER, $(hwo - 1)^7$

Energy equation:

$$
\begin{aligned}
&G_7'' + [\sigma_0F_0]G_7' \\
&\quad = (m_1m_2m_3)[-7\ {}_1G_1\ {}_1G_6'' - 35\ {}_7G_3\ {}_1G_4'' - 35\ {}_1G_4\ {}_7G_3'' - 7\ {}_1G_6\ {}_1G_1'' \\
&\qquad - 70\ {}_7G_3'\ {}_1G_4' - 14\ {}_1G_1'\ {}_1G_6' - 21({}_1G_1)^2\ {}_1G_5'' - 105\ {}_1G_1\ {}_1G_2\ {}_1G_4'' \\
&\qquad - 105\ {}_1G_1\ {}_1G_4\ {}_1G_2'' - 42\ {}_1G_1\ {}_1G_5\ {}_1G_1'' - 210\ {}_1G_1\ {}_1G_2'\ {}_1G_4' \\
&\qquad - 84\ {}_1G_1\ {}_1G_1'\ {}_1G_5' - 105\ {}_1G_2\ {}_1G_4\ {}_1G_1'' - 210\ {}_1G_2\ {}_1G_1'\ {}_1G_4' \\
&\qquad - 210\ {}_1G_4\ {}_1G_1'\ {}_1G_2' - 42\ {}_1G_5({}_1G_1')^2 - 35({}_1G_1)^3\ {}_2G_4'' \\
&\qquad - 210({}_1G_1)^2\ {}_1G_2\ {}_7G_3'' - 210({}_1G_1)^2\ {}_7G_3\ {}_1G_2'' - 105({}_1G_1)^2\ {}_2G_4\ {}_1G_1'' \\
&\qquad - 420({}_1G_1)^2\ {}_1G_2'\ {}_7G_3' - 210({}_1G_1)^2\ {}_1G_1'\ {}_2G_4' - 420\ {}_1G_1\ {}_1G_2\ {}_7G_3\ {}_1G_1'' \\
&\qquad - 840\ {}_1G_1\ {}_1G_2\ {}_1G_1'\ {}_7G_3' - 840\ {}_1G_1\ {}_7G_3\ {}_1G_1'\ {}_1G_2' - 210\ {}_1G_1\ {}_2G_4({}_1G_1')^2 \\
&\qquad - 420\ {}_1G_2\ {}_7G_3({}_1G_1')^2 - \sigma_0(7F_0\ {}_1G_6' + 21F_0\ {}_1G_5' + 35F_0\ {}_2G_4')] \\
&\quad + (\delta_1\delta_2\delta_3)[-\sigma_0(35\ {}_3G_4'\ {}_8F_3 + 35\ {}_8G_3'\ {}_2F_4)] + (m_2m_3\sigma_1)[\qquad]
\end{aligned}
\tag{B24}
$$

$$
G_7 = (m_1m_2m_3)_1G_7 + (\delta_1\delta_2\delta_3)_2G_7 + (m_2m_3\sigma_1)_aG_7 \tag{B25}
$$

Momentum equation:

$$F_7''' + [F_o]F_7'' - [F_o']F_7' + [F_o'']F_7 = (g_1g_2g_3\sigma_1)[\quad] \tag{B26}$$

$$F_7 = (g_1g_2g_3\sigma_1)_aF_7 \tag{B27}$$

H. EIGHTH ORDER, $(hwo - 1)^8$

Energy equation:

$$G_8'' + [\sigma_oF_o]G_8' = (m_1m_2m_3\sigma_1)[\quad] \tag{B28}$$

$$G_8 = (m_1m_2m_3\sigma_1)_aG_8 \tag{B29}$$

An eighth-order momentum equation was not used as the series expansion for F, equation (5.4), was terminated with the sixth order (and the seventh-order term, $_aF_7$, was moved to the sixth order and approximated).

APPENDIX C

ASYMPTOTIC SOLUTION OF THE HOMANN EQUATION

The equations that we have solved numerically ((4.12), (4.13), and (4.14)) all have a boundary condition at infinity (4.8d,e). In solving these equations on a computing machine it is necessary to replace infinity by some large number and to set the outer boundary condition at this value of the independent variable, η. The most critical of the equations is the zero-order momentum (Homann) equation (4.12), since its solution furnishes coefficients which appear on the left sides of all the higher-order equations. The value of η at which one sets the outer boundary condition will depend on the accuracy of the solution desired. In this appendix we will form an asymptotic solution to the Homann equation for large η in order to gain an idea of the value of η at which we can assign the outer boundary condition.

A. ASYMPTOTIC SOLUTION

To form the asymptotic solution to the Homann equation (4.12) with its boundary conditions (4.8a and the left side of 4.8d), we will let:

$$F_o = \eta - f_a \tag{C1a}$$

$$F_o' = 1 - f_a' \tag{C1b}$$

$$F_o'' = -f_a'' \tag{C1c}$$

$$F_o''' = -f_a''' \tag{C1d}$$

where f_a is as defined in equation (C1a). This is a logical formulation, as we know, from the boundary conditions that $f_a(\infty)$ will be a finite number and the derivatives, $f_a'(\infty)$, $f_a''(\infty)$, and $f_a'''(\infty)$ will all vanish asymptotically.

We substitute (C1) into (4.12) and use (4.8a and the left side of 4.8d) and obtain:

$$f_a''' + \eta\left(1 - \frac{f_a}{\eta}\right)f_a'' - f_a' + \frac{1}{2}(f_a')^2 = 0 \tag{C2}$$

with the boundary conditions:

$$f_a(0) = 0 \qquad f_a'(0) = 1 \qquad f_a'(\infty) = 0 \tag{C3}$$

For large η values we see that the nonlinear terms in equation (C2) will become very small. We neglect these terms and obtain the linear equation:

$$f_{as}''' + \eta f_{as}'' - f_{as}' = 0 \tag{C4}$$

where f_{as} is defined as the solution to equation (C4). The boundary conditions are still given by (C3).

Equation (C4) is actually a second-order equation since there is no f_{as} term. It has the solution:

$$f_{as} = \sqrt{\frac{\pi}{2}} \left[\int_o^\eta \eta_1 \operatorname{erf}\left(\frac{\eta_1}{\sqrt{2}}\right) d\eta_1 + \operatorname{erf}\left(\frac{\eta}{\sqrt{2}}\right) - \frac{\eta^2}{2} \right] \tag{C5}$$

where we define:

$$\operatorname{erf}(t) = \frac{2}{\sqrt{\pi}} \int_o^t e^{-t_1^2} \, dt_1$$

$$\operatorname{erf}(\infty) = 1$$

so that:

$$\operatorname{erf}\left(\frac{\eta}{\sqrt{2}}\right) = \frac{2}{\sqrt{\pi}} \int_o^{(\eta/\sqrt{2})} e^{-t_1^2} \, dt_1$$

Then, using equation (C1a), we have for the asymptotic solution to equation (4.12):

$$(F_o)_{as} = \eta - \sqrt{\frac{\pi}{2}} \left[\int_o^\eta \eta_1 \operatorname{erf}\left(\frac{\eta_1}{\sqrt{2}}\right) d\eta_1 + \operatorname{erf}\left(\frac{\eta}{\sqrt{2}}\right) - \frac{\eta^2}{2} \right] \tag{C6a}$$

$$(F_o')_{as} = 1 - \sqrt{\frac{\pi}{2}} \, \eta \left[\operatorname{erf}\left(\frac{\eta}{\sqrt{2}}\right) - 1 \right] - e^{-(\eta^2/2)} \tag{C6b}$$

The function given by equation (C6) satisfies the original boundary conditions (4.8a and the left side of 4.8d), but it will not satisfy the Homann equation (4.12) for small η values.

B. SELECTION OF MAXIMUM VALUE OF INDEPENDENT VARIABLE

For large η values we can write:

$$(F_o')_{as} = 1 - \text{error} = 1 - f_{as}' \tag{C7}$$

Then from (C6b) or (C5)

$$\text{error} = f_{as}' = \sqrt{\frac{\pi}{2}}\,\eta\left[\operatorname{erf}\left(\frac{\eta}{\sqrt{2}}\right) - 1\right] + e^{-(\eta^2/2)} \tag{C8}$$

For large arguments we can use an asymptotic formula for erf(t) (ref. 18, p. 120):

$$\operatorname{erf}(t) = 1 - \frac{e^{-t^2}}{t\sqrt{\pi}}\left[1 - \frac{1}{2t^2} + \frac{1\cdot 3}{(2t^2)^2} - \frac{1\cdot 3\cdot 5}{(2t^2)^3} + \cdots\right] \tag{C9}$$

Then we have:

$$\operatorname{erf}\left(\frac{\eta}{\sqrt{2}}\right) = 1 - \frac{e^{-(\eta^2/2)}}{\eta\sqrt{(\pi/2)}}\left(1 - \frac{1}{\eta^2} + \frac{1\cdot 3}{\eta^4} - \frac{1\cdot 3\cdot 5}{\eta^6} + \cdots\right) \tag{C10}$$

Substituting (C10) into (C8) we have:

$$\text{error} = \frac{e^{-(\eta^2/2)}}{\eta^2}\left(1 - \frac{1\cdot 3}{\eta^2} + \frac{1\cdot 3\cdot 5}{\eta^4} - \cdots\right) \tag{C11}$$

The IBM 7090 computing machine will deliver eight significant figures (without using double precision methods). Using equation (C11) we find that the smallest integer value of η that produces a value of error $< 10^{-8}$ is $\eta = 6$. So we should expect to carry our solutions of equations out at least to $\eta = 6$. Referring to equation (C1a), we know (table I) that for large η, the value of f_a becomes approximately 0.8. Then in equation (C2) the term neglected in the coefficient

of the second term, f_a/η, would have the approximate value, $0.8/6 \approx 0.13$. This is not really negligible compared to unity. If we extend the maximum value of η to $\eta = 10$, the neglected term, f_a/η, has the value $0.8/10 \approx 0.08$. This is small but still not negligible compared to unity.

In spite of this indication, for our maximum η, we will try the value $\eta = 10$. This selection is somewhat arbitrary, but it turns out to be a satisfactory value. The important factor is to observe that all the universal functions attain values very close to their asymptotes for $\eta < 10$. This condition is seen to be satisfied in table I.

APPENDIX D

EFFECT OF VARIATION OF POLYNOMIALS APPROXIMATING THE GAS PROPERTIES

In chapter 5, section E, it is noted that there are several ways of fitting polynomials to the gas properties, m, g, and δ, and the use of different methods can result in different combinations of coefficients in the polynomials. We want to determine the effects on final answers of the use of different polynomial representations. To do this we will form polynomials with different shapes and check the effect of shape on our calculated results.

In table D-1 are tabulated a number of cases with different values of the parameters, m_i, g_i, and δ_i. These parameters were selected to give a wide variation in the $m(G)$, $g(G)$, and $\delta(G)$ functions, but each family has a common value at $G = 0$. Cases with $hwo = 0$ will show the effects of varying the shapes of the gas-property curves represented by the polynomials. For each family of polynomials the extreme spread of the curves is shown plotted in figure D-1. The polynomials not plotted fall between these extremes.

The cases tabulated in table D-1 are not intended to represent any real gas (this is done in ch. 7). These hypothetical cases are for the purpose of demonstrating the effect on our calculated results of a variation in the gas-property parameters, and to gain an idea of the requirements for accuracy of representation of gas properties by polynomials.

The values of $G'(0)$ and $F''(0)$ shown in table D-1 were calculated by our method of series expansions. It is seen that the variation in wall values of $G'(0)$ and $F''(0)$ is less than one might expect. The m_1, g_1, and δ_1 cases have been used as references to tabulate the percentage changes in $G'(0)$ and $F''(0)$ for $hwo = 0$. The percentage changes in $G'(0)$ and $F''(0)$ for the cases with $hwo = 0.2$ and 0.5 have not been tabulated but they are closer together than the $hwo = 0$ cases. The differences in $G'(0)$ and $F''(0)$ for the cases with $hwo = 0.2$ and

TABLE D-1.- EFFECT OF VARIATION OF GAS-PROPERTY POLYNOMIALS

(a) m_i cases

Case	hwo	G'(0)	F"(0)	Percent change in G'(0), from m_1 case	Percent change in F"(0), from m_1 case
$m_1 = -0.75$	0	0.4289	0.9277	0	0
	.2	.3480			
	.5	.2243			
$m_2 = 1.5$	0	.4093		-4.6	0
	.2	.3487			
	.5	.2323			
$m_3 = -4.5$	0	.3954		-7.8	0
	.2	.3506			
	.5	.2361			
m_1, m_2 $m_1 = -0.375$ $m_2 = 0.75$	0	.4193		-2.2	0
	.2	.3477			
	.5	.2279			
m_1, m_3 $m_1 = -0.375$ $m_3 = -2.25$	0	.4133		-3.6	0
	.2	.3483			
	.5	.2295			
m_2, m_3 $m_2 = 0.75$ $m_3 = -2.25$	0	.4136		-3.6	0
	.2	.3526			
	.5	.2344			
m_1, m_2, m_3 $m_1 = -0.25$ $m_2 = 0.5$ $m_3 = -1.5$	0	.4160		-3.0	0
	.2	.3495			
	.5	.2305			

TABLE D-1.- EFFECT OF VARIATION OF GAS-PROPERTY POLYNOMIALS - Continued

(b) g_i cases

Case	hwo	G'(0)	F"(0)	Percent change in G'(0), from g_1 case	Percent change in F"(0), from g_1 case
$g_1 = -1.0$	0	0.4834	0.8725	0	0
	.2	.3867	.8779		
	.5	.2417	.8877		
$g_2 = 2.0$	0	.4831	.8205	close	-6.0
	.2	.3865	.8591		
	.5	.2416	.9009		
$g_3 = -6.0$	0	.4831	.7926	close	-9.2
	.2	.3865	.8585		
	.5	.2416	.9108		
g_1, g_2 $g_1 = -0.5$ $g_2 = 1.0$	0	.4833	.8453	close	-3.1
	.2	.3866	.8641		
	.5	.2416	.8905		
g_1, g_3 $g_1 = -0.5$ $g_3 = -3.0$	0	.4833	.8334	close	-4.5
	.2	.3866	.8635		
	.5	.2416	.8948		
g_2, g_3 $g_2 = 1.0$ $g_3 = -3.0$	0	.4831	.8255	close	-5.4
	.2	.3865	.8650		
	.5	.2416	.9064		
g_1, g_2, g_3 $g_1 = -1/3$ $g_2 = 2/3$ $g_3 = -2.0$	0	.4833	.8337	close	-4.45
	.2	.3866	.8626		
	.5	.2416	.8962		

TABLE D-1.- EFFECT OF VARIATION OF GAS-PROPERTY POLYNOMIALS - Concluded

(c) δ_i cases

Case	hwo	G'(0)	F"(0)	Percent change in G'(0), from δ_1 case	Percent change in F"(0), from δ_1 case
$\delta_1 = 1.0$	0	0.4405	0.5585	0	0
	.2	.3601	.6373		
	.5	.2317	.7505		
δ_1, δ_2	0	.4458	.5796	1.2	3.8
$\delta_1 = 0.8$	.2	.3637	.6624		
$\delta_2 = -0.4$	.5	.2333	.7734		
δ_1, δ_3	0	.4472	.5918	1.5	6.0
$\delta_1 = 0.8$	.2	.3647	.6756		
$\delta_3 = 1.2$	.5	.2337	.7816		
$\delta_1, \delta_2, \delta_3$	0	.4458	.5857	1.2	4.9
$\delta_1 = 0.8$	.2	.3641	.6690		
$\delta_2 = -0.2$	.5	.2335	.7775		
$\delta_3 = 0.6$					

NOTE: $\sigma_{wo} = \sigma_o = 0.75$; parameters not listed are zero for a particular case.

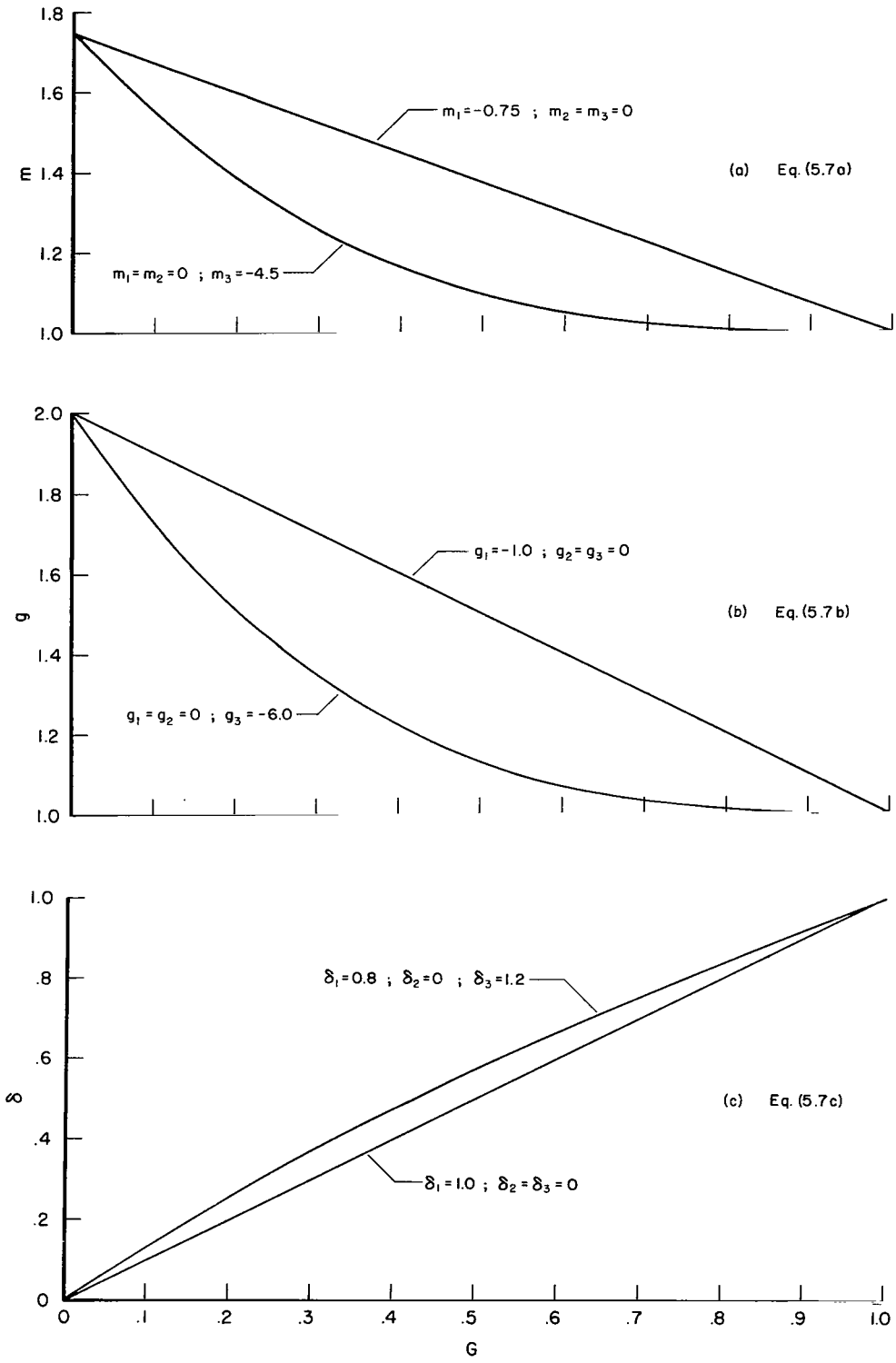


Figure D-1.- Variation of gas-property polynomials.

0.5 are due not only to differences in the shapes of the gas-property curves, but also to the different values of the gas properties at the wall.

The effect of varying the gas-property polynomials is to change the coefficients of the universal functions in equations (5.3) and (5.4). The coefficient of one universal function will be increased at the expense of another, but these universal functions have approximately the same shape and they have the same outer boundary conditions (fig. 2). The greatest effect of a variation in gas-property polynomials is on the wall derivatives as tabulated in table D-1. With this type of variation, if the wall derivatives are close in value, the functions $G(\eta)$ and $F'(\eta)$ have almost the same values through the boundary layer.

It is easy to make the calculations to show the effects on $G'(0)$ and $F''(0)$ of a variation of the gas-property parameters. One simply inserts different values of the parameters in equations (5.5) and (5.6). Combinations of the parameters can be adjusted in such a way as to maintain the wall values of the gas properties unchanged (table D-1 with hwo = 0), or the parameters can be adjusted to change the shapes of the gas-property curves and the wall values as well (table D-1 with hwo = 0.2 and 0.5). This latter type of variation is of interest as it may be desired to use one writing of the gas-property polynomials for a number of cases with different hwo values, and there may be values of hwo at which the fit of the polynomials to the actual gas properties is less than ideal. The cases in table D-1 with hwo = 0.2 and 0.5 show only a small variation in $G'(0)$ and $F''(0)$. For these cases the extremes of the spread of gas properties at the wall are seen in figure D-1 (at G = 0.2 and 0.5).

We can conclude that our calculated results are not overly sensitive to the selection of the gas-property polynomials used. Equations (5.3) through (5.6) have large terms which do not depend on the gas properties. Also the sensitivity to the polynomials selected decreases to zero as hwo approaches unity.

The spreads of the variations in the shapes of the gas-property curves shown in figure D-1 are quite extreme. A reasonable attempt to match gas properties with polynomials should fall well within this

spread. The matching can be made very close for flight in the earth's atmosphere at free-stream Mach numbers up to about 5. The matching becomes progressively more difficult with increasing Mach numbers. Some examples of this are shown in chapter 7. While it is apparently not crucial, it seems desirable to correctly match the wall values of the gas properties when it is convenient to do so.

APPENDIX E

SUMMARY OF WORKING FORMULAS VALID IN THE VICINITY OF THE STAGNATION POINT

The stagnation-region working formulas are listed in table E-1. Most of the formulas have been machine programmed as indicated by asterisk symbols in the table. Parameter limits used for the series truncations are shown. Parameter values selected for use should generally be kept within these limits. The limit, hwo equal to zero, is, obviously, not physically realizable.

TABLE E-1.- STAGNATION-REGION WORKING FORMULAS

Quantity be determined	Equation number
G, G', G''	5.3*
F, F', F'', F'''	5.4*
$G'(0)$	5.5*
$F''(0)$	5.6*
ξ or x	6.3
a_1 or $(du_e/d_x)_{(x=0)}$	6.4, 6.5, or 6.6
y	6.8*
y_{AP}	6.12*
y_P	6.12* ($\delta_1 = 1$)
ρv	6.14*
ρ	6.15*
ρ_{AP}	6.16*
ρ_P	6.16* ($\delta_1 = 1$)
v	6.17 or 6.18*
v_{AP}	6.18 and 6.19*
v_P	6.18 and 6.19* ($\delta_1 = 1$)
δ^*	6.23*
δ^*_{AP}	6.25*
δ^*_P	6.25* ($\delta_1 = 1$)

TABLE E-1.- STAGNATION-REGION WORKING FORMULAS - Concluded.

Quantity to be determined	Equation number
$C_{fe}\sqrt{Re}$	6.41*
$C_{fw}\sqrt{Rw}$	6.42*
Q_w	6.56*
$Nu_w/\sqrt{Rw}$	6.57*
$St_w\sqrt{Rw}$	6.58*
*Formulas are machine programmed	

Basic formulas

$$G = h_o = H_o/H_{st}$$

$$F' = f_o' = u/u_e$$

$$u_e \approx (du_e/d_x)_{(x=o)}x$$

$$u_e \approx \sqrt{2H_{st}}\, a_1\xi$$

Gas-property polynomials

$$m(G) = \frac{\rho\mu/\sigma}{(\rho\mu/\sigma)_{so}} = 1 + m_1(G - 1) + \frac{m_2}{2}(G - 1)^2 + \frac{m_3}{6}(G - 1)^3 \quad (5.7a)$$

$$g(G) = \frac{\rho\mu}{(\rho\mu)_{so}} = 1 + g_1(G - 1) + \frac{g_2}{2}(G - 1)^2 + \frac{g_3}{6}(G - 1)^3 \quad (5.7b)$$

$$\delta(G) = \frac{\rho_{so}}{\rho} = 1 + \delta_1(G - 1) + \frac{\delta_2}{2}(G - 1)^2 + \frac{\delta_3}{6}(G - 1)^3 \quad (5.7c)$$

Parameter limits used for series truncations

$0 \le hwo \le 1$	$0 \le g_2(hwo - 1)^2 \le 2.0$
$0 \le m_1(hwo - 1) \le 1.0$	$-0.4 \le \delta_2(hwo - 1)^2 \le 0$
$0 \le g_1(hwo - 1) \le 1.5$	$0 \le m_3(hwo - 1)^3 \le 4.5$
$-1.0 \le \delta_1(hwo - 1) \le 0$	$0 \le g_3(hwo - 1)^3 \le 6.0$
$0 \le m_2(hwo - 1)^2 \le 1.5$	$-1.2 \le \delta_3(hwo - 1)^3 \le 0$

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TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$

η	F_0	F_0'	F_0''	F_0'''	${}_1F_1$ (g_1)	${}_1F_1'$	${}_1F_1''$	${}_1F_1'''$
0.	0.	0.	0.92768	-0.50000	0.	0.	-0.12761	0.44818
0.01000	0.00005	0.00925	0.92268	-0.50000	-0.00001	-0.00125	-0.12315	0.44335
0.02000	0.00018	0.01845	0.91768	-0.50000	-0.00002	-0.00246	-0.11874	0.43852
0.03000	0.00042	0.02761	0.91268	-0.50000	-0.00006	-0.00363	-0.11438	0.43368
0.04000	0.00074	0.03671	0.90768	-0.50000	-0.00010	-0.00475	-0.11007	0.42885
0.05000	0.00115	0.04576	0.90268	-0.49999	-0.00015	-0.00583	-0.10580	0.42401
0.06000	0.00165	0.05476	0.89768	-0.49998	-0.00021	-0.00687	-0.10159	0.41917
0.07000	0.00224	0.06371	0.89268	-0.49997	-0.00029	-0.00786	-0.09742	0.41432
0.08000	0.00293	0.07261	0.88768	-0.49996	-0.00037	-0.00882	-0.09330	0.40947
0.09000	0.00370	0.08147	0.88268	-0.49994	-0.00046	-0.00973	-0.08923	0.40461
0.10000	0.00456	0.09027	0.87768	-0.49992	-0.00057	-0.01060	-0.08521	0.39975
0.11000	0.00550	0.09902	0.87268	-0.49990	-0.00068	-0.01143	-0.08123	0.39489
0.12000	0.00654	0.10772	0.86768	-0.49987	-0.00079	-0.01223	-0.07731	0.39001
0.13000	0.00766	0.11637	0.86269	-0.49983	-0.00092	-0.01298	-0.07343	0.38513
0.14000	0.00886	0.12498	0.85769	-0.49979	-0.00105	-0.01369	-0.06961	0.38024
0.15000	0.01016	0.13353	0.85269	-0.49974	-0.00119	-0.01437	-0.06583	0.37535
0.20000	0.01789	0.17554	0.82771	-0.49940	-0.00199	-0.01720	-0.04768	0.35076
0.25000	0.02769	0.21630	0.80275	-0.49883	-0.00290	-0.01916	-0.03076	0.32596
0.30000	0.03950	0.25581	0.77783	-0.49800	-0.00389	-0.02030	-0.01508	0.30096
0.35000	0.05325	0.29408	0.75296	-0.49685	-0.00492	-0.02069	-0.00066	0.27579
0.40000	0.06888	0.33111	0.72815	-0.49534	-0.00595	-0.02039	0.01249	0.25049
0.45000	0.08634	0.36690	0.70343	-0.49343	-0.00695	-0.01946	0.02438	0.22512
0.50000	0.10555	0.40146	0.67882	-0.49107	-0.00788	-0.01797	0.03501	0.19977
0.55000	0.12646	0.43478	0.65433	-0.48823	-0.00873	-0.01598	0.04436	0.17453
0.60000	0.14901	0.46689	0.63000	-0.48488	-0.00947	-0.01355	0.05246	0.14952
0.65000	0.17313	0.49779	0.60585	-0.48100	-0.01008	-0.01075	0.05932	0.12486
0.70000	0.19877	0.52748	0.58191	-0.47655	-0.01054	-0.00764	0.06496	0.10068
0.80000	0.25435	0.58330	0.53477	-0.46590	-0.01097	-0.00072	0.07268	0.05432
0.90000	0.31528	0.63447	0.48881	-0.45283	-0.01067	0.00674	0.07594	0.01158
1.00000	0.38109	0.68112	0.44428	-0.43735	-0.00962	0.01433	0.07516	-0.02645
1.10000	0.45135	0.72339	0.40142	-0.41954	-0.00781	0.02166	0.07084	-0.05880
1.20000	0.52563	0.76146	0.36045	-0.39955	-0.00530	0.02840	0.06361	-0.08472
1.40000	0.68461	0.82585	0.28498	-0.35408	0.00152	0.03918	0.04310	-0.11574
1.60000	0.85503	0.87610	0.21915	-0.30361	0.01006	0.04541	0.01912	-0.11982
1.80000	1.03424	0.91420	0.16365	-0.25137	0.01937	0.04693	-0.00338	-0.10226
2.00000	1.22004	0.94225	0.11850	-0.20066	0.02856	0.04440	-0.02091	-0.07167
2.20000	1.41061	0.96225	0.08310	-0.15425	0.03694	0.03901	-0.03180	-0.03734
2.40000	1.60453	0.97607	0.05638	-0.11411	0.04407	0.03212	-0.03611	-0.00689
2.60000	1.80073	0.98529	0.03697	-0.08118	0.04977	0.02492	-0.03512	0.01518
2.80000	1.99843	0.99125	0.02342	-0.05552	0.05407	0.01830	-0.03069	0.02764
3.00000	2.19708	0.99496	0.01432	-0.03650	0.05716	0.01275	-0.02463	0.03171
3.20000	2.39631	0.99719	0.00845	-0.02305	0.05926	0.00846	-0.01840	0.02989
3.40000	2.59589	0.99849	0.00481	-0.01399	0.06062	0.00534	-0.01289	0.02487
3.60000	2.79567	0.99921	0.00264	-0.00816	0.06146	0.00322	-0.00851	0.01883
3.80000	2.99556	0.99960	0.00139	-0.00457	0.06196	0.00186	-0.00533	0.01320
4.00000	3.19550	0.99981	0.00071	-0.00246	0.06224	0.00103	-0.00316	0.00864
4.20000	3.39547	0.99991	0.00035	-0.00127	0.06239	0.00054	-0.00179	0.00532
4.40000	3.59546	0.99996	0.00016	-0.00063	0.06247	0.00027	-0.00096	0.00309
4.60000	3.79545	0.99998	0.00007	-0.00030	0.06251	0.00013	-0.00049	0.00170
4.80000	3.99545	0.99999	0.00003	-0.00014	0.06253	0.00006	-0.00024	0.00089
5.00000	4.19545	1.00000	0.00001	-0.00006	0.06254	0.00003	-0.00011	0.00044
5.20000	4.39545	1.00000	0.00001	-0.00003	0.06254	0.00001	-0.00005	0.00021
5.40000	4.59545	1.00000	0.00000	-0.00001	0.06254	0.00000	-0.00002	0.00010
5.60000	4.79545	1.00000	0.00000	-0.00000	0.06254	0.00000	-0.00001	0.00004
5.80000	4.99545	1.00000	0.00000	-0.00000	0.06254	0.00000	-0.00000	0.00002
6.00000	5.19545	1.00000	0.00000	-0.00000	0.06254	0.00000	-0.00000	0.00001
6.20000	5.39545	1.00000	0.00000	-0.00000	0.06254	0.00000	-0.00000	0.00000
6.40000	5.59545	1.00000	-0.00000	-0.00000	0.06254	0.00000	-0.00000	0.00000
6.60000	5.79545	1.00000	-0.00000	-0.00000	0.06254	0.00000	-0.00000	0.00000
6.80000	5.99545	1.00000	-0.00000	-0.00000	0.06254	0.00000	-0.00000	0.00000
7.00000	6.19545	1.00000	-0.00000	0.00000	0.06254	0.00000	-0.00000	0.00000
7.20000	6.39545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	0.00000
7.40000	6.59545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	-0.00000
7.60000	6.79545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	-0.00000
7.80000	6.99545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	-0.00000
8.00000	7.19545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	-0.00000
8.20000	7.39545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	-0.00000
8.40000	7.59545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	-0.00000
8.60000	7.79545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	0.00000
8.80000	7.99545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	0.00000
9.00000	8.19545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	-0.00000
9.20000	8.39545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	0.00000
9.40000	8.59545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	0.00000
9.60000	8.79545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	0.00000
9.80000	8.99545	1.00000	-0.00000	0.00000	0.06254	0.00000	0.00000	0.00000
10.00000	9.19545	1.00000	-0.00000	-0.00000	0.06254	0.00000	0.00000	0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_2F_1$ (δ_1)	$_2F_1'$	$_2F_1''$	$_2F_1'''$	$_1F_2$ (g_1^2)	$_1F_2'$	$_1F_2''$	$_1F_2'''$
0.	0.	0.	0.34254	-0.50000	0.	0.	0.23707	-1.01873
0.01000	0.00002	0.00340	0.33755	-0.49758	0.00001	0.00232	0.22698	-1.00048
0.02000	0.00007	0.00675	0.33259	-0.49517	0.00005	0.00454	0.21706	-0.98238
0.03000	0.00015	0.01005	0.32765	-0.49275	0.00010	0.00666	0.20733	-0.96441
0.04000	0.00027	0.01330	0.32273	-0.49033	0.00018	0.00869	0.19778	-0.94657
0.05000	0.00042	0.01651	0.31784	-0.48791	0.00028	0.01062	0.18840	-0.92887
0.06000	0.00060	0.01966	0.31298	-0.48548	0.00039	0.01246	0.17920	-0.91130
0.07000	0.00081	0.02277	0.30813	-0.48306	0.00052	0.01420	0.17017	-0.89387
0.08000	0.00105	0.02582	0.30332	-0.48062	0.00067	0.01586	0.16132	-0.87656
0.09000	0.00133	0.02883	0.29852	-0.47819	0.00084	0.01743	0.15264	-0.85938
0.10000	0.00163	0.03179	0.29375	-0.47574	0.00102	0.01891	0.14413	-0.84233
0.11000	0.00196	0.03471	0.28901	-0.47330	0.00122	0.02031	0.13579	-0.82541
0.12000	0.00232	0.03757	0.28429	-0.47084	0.00143	0.02163	0.12762	-0.80862
0.13000	0.00271	0.04039	0.27959	-0.46838	0.00165	0.02287	0.11962	-0.79195
0.14000	0.00313	0.04317	0.27492	-0.46591	0.00189	0.02402	0.11178	-0.77541
0.15000	0.00358	0.04589	0.27027	-0.46344	0.00213	0.02510	0.10411	-0.75900
0.20000	0.00620	0.05883	0.24741	-0.45093	0.00350	0.02939	0.06818	-0.67885
0.25000	0.00944	0.07064	0.22518	-0.43817	0.00504	0.03199	0.03617	-0.60193
0.30000	0.01325	0.08136	0.20360	-0.42512	0.00668	0.03307	0.00793	-0.52831
0.35000	0.01756	0.09102	0.18268	-0.41174	0.00833	0.03284	-0.01672	-0.45814
0.40000	0.02233	0.09964	0.16243	-0.39800	0.00994	0.03146	-0.03794	-0.39158
0.45000	0.02751	0.10727	0.14288	-0.38390	0.01146	0.02910	-0.05594	-0.32878
0.50000	0.03304	0.11394	0.12405	-0.36941	0.01284	0.02592	-0.07089	-0.26992
0.55000	0.03889	0.11969	0.10595	-0.35455	0.01404	0.02206	-0.08300	-0.21517
0.60000	0.04500	0.12455	0.08860	-0.33931	0.01503	0.01766	-0.09247	-0.16466
0.65000	0.05133	0.12856	0.07202	-0.32371	0.01580	0.01285	-0.09954	-0.11851
0.70000	0.05784	0.13176	0.05623	-0.30778	0.01631	0.00774	-0.10440	-0.07678
0.80000	0.07125	0.13590	0.02708	-0.27505	0.01656	-0.00296	-0.10842	-0.00665
0.90000	0.08493	0.13729	0.00125	-0.24146	0.01572	-0.01374	-0.10631	0.04613
1.00000	0.09862	0.13626	-0.02119	-0.20744	0.01382	-0.02407	-0.09973	0.08282
1.10000	0.11211	0.13316	-0.04024	-0.17351	0.01093	-0.03359	-0.09021	0.10542
1.20000	0.12520	0.12833	-0.05592	-0.14021	0.00714	-0.04206	-0.07903	0.11653
1.40000	0.14958	0.11477	-0.07760	-0.07779	-0.00269	-0.05550	-0.05541	0.11589
1.60000	0.17090	0.09807	-0.08764	-0.02438	-0.01475	-0.06435	-0.03352	0.10238
1.80000	0.18874	0.08035	-0.08816	0.01702	-0.02816	-0.06909	-0.01438	0.08967
2.00000	0.20308	0.06327	-0.08172	0.04513	-0.04215	-0.07024	0.00264	0.08100
2.20000	0.21417	0.04795	-0.07096	0.06048	-0.05604	-0.06814	0.01803	0.07244
2.40000	0.22242	0.03502	-0.05824	0.06508	-0.06921	-0.06317	0.03127	0.05886
2.60000	0.22835	0.02466	-0.04545	0.06180	-0.08115	-0.05586	0.04111	0.03851
2.80000	0.23245	0.01675	-0.03385	0.05368	-0.09146	-0.04703	0.04639	0.01401
3.00000	0.23519	0.01099	-0.02412	0.04343	-0.09993	-0.03764	0.04678	-0.00956
3.20000	0.23696	0.00697	-0.01648	0.03306	-0.10654	-0.02860	0.04295	-0.02749
3.40000	0.23807	0.00427	-0.01082	0.02384	-0.11144	-0.02064	0.03632	-0.03740
3.60000	0.23873	0.00253	-0.00683	0.01636	-0.11489	-0.01415	0.02851	-0.03951
3.80000	0.23912	0.00145	-0.00415	0.01071	-0.11721	-0.00923	0.02090	-0.03586
4.00000	0.23934	0.00080	-0.00243	0.00672	-0.11868	-0.00572	0.01437	-0.02911
4.20000	0.23946	0.00043	-0.00138	0.00404	-0.11957	-0.00338	0.00931	-0.02158
4.40000	0.23952	0.00023	-0.00075	0.00234	-0.12009	-0.00190	0.00569	-0.01479
4.60000	0.23956	0.00011	-0.00040	0.00130	-0.12037	-0.00102	0.00329	-0.00944
4.80000	0.23957	0.00006	-0.00020	0.00070	-0.12052	-0.00052	0.00181	-0.00565
5.00000	0.23958	0.00003	-0.00010	0.00036	-0.12060	-0.00026	0.00094	-0.00319
5.20000	0.23958	0.00001	-0.00005	0.00018	-0.12063	-0.00012	0.00047	-0.00170
5.40000	0.23959	0.00001	-0.00002	0.00009	-0.12065	-0.00005	0.00022	-0.00086
5.60000	0.23959	0.00000	-0.00001	0.00004	-0.12066	-0.00002	0.00010	-0.00041
5.80000	0.23959	0.00000	-0.00000	0.00002	-0.12066	-0.00001	0.00004	-0.00019
6.00000	0.23959	0.00000	-0.00000	0.00001	-0.12066	-0.00000	0.00002	-0.00008
6.20000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	-0.00000	0.00001	-0.00003
6.40000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	-0.00000	0.00000	-0.00001
6.60000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	-0.00000	0.00000	-0.00000
6.80000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	-0.00000	0.00000	-0.00000
7.00000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
7.20000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
7.40000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
7.60000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
7.80000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
8.00000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
8.20000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
8.40000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	-0.00000
8.60000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	0.00000
8.80000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	-0.00000	0.00000
9.00000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	-0.00000	0.00000
9.20000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	0.00000
9.40000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	0.00000	0.00000
9.60000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	-0.00000	0.00000
9.80000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	-0.00000	0.00000
10.00000	0.23959	-0.00000	-0.00000	0.00000	-0.12066	0.00000	-0.00000	0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_2F_2$ $(g_1\delta_1)$	$_2F_2'$	$_2F_2''$	$_2F_2'''$	$_3F_2$ (δ_1^2)	$_3F_2'$	$_3F_2''$	$_3F_2'''$
0.	0.	0.	-0.00472	0.39757	0.	0.	-0.04195	0.
0.01000	-0.00000	-0.00003	-0.00079	0.38723	-0.00000	-0.00042	-0.04195	0.00036
0.02000	-0.00000	-0.00002	0.00303	0.37696	-0.00001	-0.00084	-0.04194	0.00072
0.03000	-0.00000	0.00003	0.00675	0.36675	-0.00002	-0.00126	-0.04193	0.00108
0.04000	0.00000	0.00012	0.01036	0.35662	-0.00003	-0.00168	-0.04192	0.00144
0.05000	0.00000	0.00024	0.01388	0.34654	-0.00005	-0.00210	-0.04190	0.00180
0.06000	0.00001	0.00040	0.01730	0.33654	-0.00008	-0.00252	-0.04188	0.00216
0.07000	0.00001	0.00059	0.02061	0.32660	-0.00010	-0.00293	-0.04186	0.00253
0.08000	0.00002	0.00081	0.02383	0.31672	-0.00013	-0.00335	-0.04183	0.00290
0.09000	0.00003	0.00106	0.02695	0.30690	-0.00017	-0.00377	-0.04180	0.00327
0.10000	0.00004	0.00135	0.02997	0.29715	-0.00021	-0.00419	-0.04177	0.00364
0.11000	0.00005	0.00166	0.03289	0.28746	-0.00025	-0.00461	-0.04173	0.00401
0.12000	0.00007	0.00200	0.03571	0.27783	-0.00030	-0.00502	-0.04169	0.00439
0.13000	0.00009	0.00237	0.03845	0.26826	-0.00035	-0.00544	-0.04164	0.00477
0.14000	0.00012	0.00277	0.04108	0.25875	-0.00041	-0.00586	-0.04159	0.00515
0.15000	0.00015	0.00320	0.04362	0.24931	-0.00047	-0.00627	-0.04154	0.00554
0.20000	0.00037	0.00567	0.05492	0.20303	-0.00084	-0.00834	-0.04121	0.00753
0.25000	0.00072	0.00865	0.06395	0.15836	-0.00130	-0.01039	-0.04078	0.00962
0.30000	0.00124	0.01203	0.07079	0.11537	-0.00188	-0.01242	-0.04025	0.01182
0.35000	0.00193	0.01569	0.07552	0.07422	-0.00255	-0.01441	-0.03960	0.01413
0.40000	0.00281	0.01955	0.07824	0.03505	-0.00332	-0.01638	-0.03883	0.01656
0.45000	0.00389	0.02349	0.07906	-0.00194	-0.00418	-0.01830	-0.03794	0.01908
0.50000	0.00516	0.02742	0.07809	-0.03655	-0.00514	-0.02017	-0.03692	0.02169
0.55000	0.00663	0.03127	0.07545	-0.06858	-0.00620	-0.02199	-0.03577	0.02436
0.60000	0.00828	0.03494	0.07128	-0.09780	-0.00734	-0.02374	-0.03449	0.02708
0.65000	0.01012	0.03837	0.06572	-0.12402	-0.00857	-0.02543	-0.03306	0.02981
0.70000	0.01212	0.04149	0.05893	-0.14705	-0.00988	-0.02705	-0.03151	0.03252
0.80000	0.01653	0.04658	0.04231	-0.18290	-0.01274	-0.03003	-0.02799	0.03775
0.90000	0.02137	0.04986	0.02283	-0.20444	-0.01588	-0.03263	-0.02397	0.04249
1.00000	0.02644	0.05110	0.00191	-0.21145	-0.01925	-0.03481	-0.01952	0.04644
1.10000	0.03152	0.05024	-0.01900	-0.20446	-0.02282	-0.03652	-0.01472	0.04935
1.20000	0.03642	0.04735	-0.03856	-0.18486	-0.02654	-0.03774	-0.00969	0.05098
1.40000	0.04489	0.03634	-0.06926	-0.11694	-0.03421	-0.03866	0.00050	0.04993
1.60000	0.05065	0.02071	-0.08411	-0.03065	-0.04187	-0.03759	0.00990	0.04316
1.80000	0.05309	0.00384	-0.08196	0.04918	-0.04914	-0.03482	0.01746	0.03190
2.00000	0.05231	-0.01115	-0.06614	0.10377	-0.05571	-0.03078	0.02250	0.01832
2.20000	0.04890	-0.02211	-0.04269	0.12518	-0.06140	-0.02600	0.02480	0.00489
2.40000	0.04380	-0.02816	-0.01809	0.11653	-0.06610	-0.02103	0.02461	-0.00633
2.60000	0.03795	-0.02961	0.00261	0.08819	-0.06983	-0.01629	0.02250	-0.01411
2.80000	0.03219	-0.02756	0.01670	0.05255	-0.07265	-0.01210	0.01921	-0.01819
3.00000	0.02707	-0.02340	0.02383	0.01978	-0.07472	-0.00864	0.01544	-0.01904
3.20000	0.02288	-0.01841	0.02520	-0.00432	-0.07616	-0.00592	0.01175	-0.01757
3.40000	0.01969	-0.01357	0.02278	-0.01826	-0.07713	-0.00391	0.00850	-0.01479
3.60000	0.01741	-0.00942	0.01848	-0.02349	-0.07776	-0.00248	0.00586	-0.01155
3.80000	0.01586	-0.00620	0.01378	-0.02280	-0.07816	-0.00152	0.00387	-0.00846
4.00000	0.01487	-0.00388	0.00957	-0.01900	-0.07839	-0.00090	0.00244	-0.00586
4.20000	0.01426	-0.00231	0.00624	-0.01424	-0.07853	-0.00051	0.00148	-0.00385
4.40000	0.01390	-0.00132	0.00385	-0.00982	-0.07861	-0.00028	0.00086	-0.00241
4.60000	0.01370	-0.00072	0.00225	-0.00632	-0.07865	-0.00015	0.00049	-0.00145
4.80000	0.01360	-0.00038	0.00126	-0.00382	-0.07867	-0.00008	0.00026	-0.00083
5.00000	0.01354	-0.00019	0.00067	-0.00218	-0.07868	-0.00004	0.00014	-0.00046
5.20000	0.01352	-0.00009	0.00034	-0.00118	-0.07869	-0.00002	0.00007	-0.00024
5.40000	0.01350	-0.00004	0.00017	-0.00061	-0.07869	-0.00001	0.00003	-0.00012
5.60000	0.01350	-0.00002	0.00008	-0.00030	-0.07869	-0.00000	0.00002	-0.00006
5.80000	0.01350	-0.00001	0.00004	-0.00014	-0.07869	-0.00000	0.00001	-0.00003
6.00000	0.01349	-0.00000	0.00002	-0.00007	-0.07869	-0.00000	0.00000	-0.00001
6.20000	0.01349	-0.00000	0.00001	-0.00003	-0.07869	-0.00000	0.00000	-0.00001
6.40000	0.01349	-0.00000	0.00000	-0.00001	-0.07869	-0.00000	0.00000	-0.00000
6.60000	0.01349	-0.00000	0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
6.80000	0.01349	-0.00000	0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
7.00000	0.01349	-0.00000	0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
7.20000	0.01349	-0.00000	0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
7.40000	0.01349	-0.00000	-0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
7.60000	0.01349	-0.00000	-0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
7.80000	0.01349	-0.00000	-0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
8.00000	0.01349	-0.00000	-0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
8.20000	0.01349	-0.00000	-0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
8.40000	0.01349	-0.00000	-0.00000	-0.00000	-0.07869	0.00000	0.00000	-0.00000
8.60000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
8.80000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
9.00000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
9.20000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
9.40000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
9.60000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
9.80000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000
10.00000	0.01349	-0.00000	-0.00000	0.00000	-0.07869	0.00000	0.00000	-0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_4F_2$ (g_2)	${}_4F_2'$	${}_4F_2''$	${}_4F_2'''$	${}_5F_2$ (δ_2)	${}_5F_2'$	${}_5F_2''$	${}_5F_2'''$
0.	0.	0.	-0.20073	0.89636	0.	0.	0.25133	-0.50000
0.01000	-0.00001	-0.00196	-0.19184	0.88240	0.00001	0.00249	0.24636	-0.49518
0.02000	-0.00004	-0.00384	-0.18308	0.86851	0.00005	0.00493	0.24143	-0.49038
0.03000	-0.00009	-0.00562	-0.17447	0.85469	0.00011	0.00732	0.23655	-0.48561
0.04000	-0.00015	-0.00733	-0.16599	0.84093	0.00020	0.00966	0.23172	-0.48086
0.05000	-0.00023	-0.00895	-0.15765	0.82724	0.00030	0.01195	0.22693	-0.47612
0.06000	-0.00033	-0.01048	-0.14944	0.81361	0.00043	0.01420	0.22219	-0.47141
0.07000	-0.00044	-0.01193	-0.14138	0.80005	0.00059	0.01640	0.21750	-0.46672
0.08000	-0.00057	-0.01331	-0.13344	0.78654	0.00076	0.01855	0.21286	-0.46205
0.09000	-0.00071	-0.01460	-0.12565	0.77310	0.00096	0.02065	0.20826	-0.45740
0.10000	-0.00086	-0.01582	-0.11798	0.75972	0.00118	0.02271	0.20371	-0.45277
0.11000	-0.00102	-0.01696	-0.11045	0.74639	0.00141	0.02473	0.19921	-0.44815
0.12000	-0.00120	-0.01803	-0.10305	0.73313	0.00167	0.02670	0.19475	-0.44355
0.13000	-0.00138	-0.01903	-0.09579	0.71992	0.00195	0.02862	0.19034	-0.43898
0.14000	-0.00158	-0.01995	-0.08865	0.70677	0.00224	0.03050	0.18597	-0.43441
0.15000	-0.00178	-0.02080	-0.08165	0.69368	0.00256	0.03234	0.18165	-0.42987
0.20000	-0.00291	-0.02404	-0.04859	0.62909	0.00439	0.04090	0.16072	-0.40739
0.25000	-0.00416	-0.02571	-0.01872	0.56594	0.00663	0.04843	0.14090	-0.38531
0.30000	-0.00546	-0.02597	0.00803	0.50428	0.00922	0.05500	0.12218	-0.36359
0.35000	-0.00674	-0.02496	0.03173	0.44419	0.01211	0.06067	0.10454	-0.34222
0.40000	-0.00794	-0.02284	0.05248	0.38580	0.01527	0.06548	0.08795	-0.32120
0.45000	-0.00900	-0.01976	0.07034	0.32924	0.01865	0.06948	0.07241	-0.30053
0.50000	-0.00990	-0.01585	0.08543	0.27468	0.02221	0.07273	0.05789	-0.28023
0.55000	-0.01058	-0.01126	0.09785	0.22229	0.02591	0.07529	0.04438	-0.26030
0.60000	-0.01101	-0.00611	0.10770	0.17228	0.02972	0.07719	0.03186	-0.24078
0.65000	-0.01118	-0.00053	0.11512	0.12482	0.03362	0.07849	0.02030	-0.22168
0.70000	-0.01106	0.00536	0.12023	0.08011	0.03756	0.07923	0.00968	-0.20303
0.80000	-0.00992	0.01765	0.12411	-0.00037	0.04550	0.07925	-0.00881	-0.16724
0.90000	-0.00753	0.02994	0.12059	-0.06794	0.05336	0.07759	-0.02384	-0.13369
1.00000	-0.00395	0.04156	0.11099	-0.12177	0.06098	0.07459	-0.03564	-0.10264
1.10000	0.00074	0.05198	0.09670	-0.16154	0.06824	0.07056	-0.04446	-0.07438
1.20000	0.00639	0.06079	0.07914	-0.18745	0.07506	0.06578	-0.05061	-0.04910
1.40000	0.01987	0.07270	0.03947	-0.20143	0.08716	0.05498	-0.05611	-0.00805
1.60000	0.03494	0.07668	0.00123	-0.17557	0.09703	0.04380	-0.05469	0.02020
1.80000	0.05008	0.07372	-0.02919	-0.12624	0.10473	0.03340	-0.04882	0.03664
2.00000	0.06409	0.06573	-0.04880	-0.06985	0.11049	0.02442	-0.04068	0.04331
2.20000	0.07619	0.05493	-0.05753	-0.01921	0.11461	0.01716	-0.03197	0.04285
2.40000	0.08601	0.04332	-0.05737	0.01832	0.11746	0.01159	-0.02384	0.03794
2.60000	0.09356	0.03238	-0.05123	0.04059	0.11935	0.00754	-0.01693	0.03093
2.80000	0.09907	0.02303	-0.04204	0.04935	0.12056	0.00472	-0.01149	0.02356
3.00000	0.10290	0.01561	-0.03214	0.04830	0.12130	0.00285	-0.00746	0.01690
3.20000	0.10545	0.01011	-0.02310	0.04152	0.12175	0.00166	-0.00464	0.01149
3.40000	0.10706	0.00626	-0.01569	0.03247	0.12200	0.00093	-0.00277	0.00743
3.60000	0.10804	0.00371	-0.01010	0.02351	0.12214	0.00050	-0.00159	0.00458
3.80000	0.10861	0.00211	-0.00619	0.01592	0.12221	0.00026	-0.00088	0.00270
4.00000	0.10892	0.00115	-0.00361	0.01015	0.12225	0.00013	-0.00046	0.00152
4.20000	0.10909	0.00060	-0.00201	0.00612	0.12227	0.00006	-0.00024	0.00082
4.40000	0.10918	0.00030	-0.00107	0.00350	0.12228	0.00003	-0.00012	0.00042
4.60000	0.10922	0.00014	-0.00054	0.00190	0.12228	0.00001	-0.00005	0.00021
4.80000	0.10924	0.00007	-0.00026	0.00098	0.12228	0.00001	-0.00002	0.00010
5.00000	0.10925	0.00003	-0.00012	0.00048	0.12228	0.00000	-0.00001	0.00005
5.20000	0.10926	0.00001	-0.00006	0.00023	0.12228	0.00000	-0.00000	0.00002
5.40000	0.10926	0.00001	-0.00002	0.00010	0.12228	0.00000	-0.00000	0.00001
5.60000	0.10926	0.00000	-0.00001	0.00004	0.12228	-0.00000	-0.00000	0.00000
5.80000	0.10926	0.00000	-0.00000	0.00002	0.12228	-0.00000	-0.00000	0.00000
6.00000	0.10926	0.00000	-0.00000	0.00001	0.12228	-0.00000	-0.00000	0.00000
6.20000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
6.40000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
6.60000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
6.80000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
7.00000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
7.20000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
7.40000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	0.00000
7.60000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	-0.00000
7.80000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	-0.00000
8.00000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	-0.00000
8.20000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	-0.00000
8.40000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	0.00000
8.60000	0.10926	0.00000	0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
8.80000	0.10926	0.00000	0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
9.00000	0.10926	0.00000	0.00000	-0.00000	0.12228	-0.00000	-0.00000	0.00000
9.20000	0.10926	0.00000	0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
9.40000	0.10926	0.00000	0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
9.60000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
9.80000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000
10.00000	0.10926	0.00000	-0.00000	0.00000	0.12228	-0.00000	-0.00000	0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_1F_3$ (g_1^3)	${}_1F_3'$	${}_1F_3''$	${}_1F_3'''$	${}_2F_3$ (δ_1^3)	${}_2F_3'$	${}_2F_3''$	${}_2F_3'''$
0.	0.	0.	-0.66731	3.39677	0.	0.	0.02466	-0.
0.01000	-0.00003	-0.00650	-0.63376	3.31296	0.00000	0.00025	0.02466	-0.00015
0.02000	-0.00013	-0.01268	-0.60105	3.23035	0.00000	0.00049	0.02465	-0.00031
0.03000	-0.00029	-0.01853	-0.56915	3.14893	0.00001	0.00074	0.02465	-0.00046
0.04000	-0.00050	-0.02406	-0.53807	3.06869	0.00002	0.00099	0.02464	-0.00061
0.05000	-0.00077	-0.02929	-0.50778	2.98960	0.00003	0.00123	0.02464	-0.00077
0.06000	-0.00108	-0.03422	-0.47827	2.91167	0.00004	0.00148	0.02463	-0.00092
0.07000	-0.00145	-0.03886	-0.44954	2.83487	0.00006	0.00173	0.02462	-0.00107
0.08000	-0.00186	-0.04322	-0.42157	2.75920	0.00008	0.00197	0.02461	-0.00123
0.09000	-0.00231	-0.04729	-0.39435	2.68465	0.00010	0.00222	0.02459	-0.00138
0.10000	-0.00280	-0.05110	-0.36787	2.61121	0.00012	0.00246	0.02458	-0.00154
0.11000	-0.00333	-0.05465	-0.34212	2.53887	0.00015	0.00271	0.02456	-0.00169
0.12000	-0.00390	-0.05795	-0.31709	2.46762	0.00018	0.00295	0.02455	-0.00185
0.13000	-0.00449	-0.06100	-0.29277	2.39746	0.00021	0.00320	0.02453	-0.00200
0.14000	-0.00512	-0.06381	-0.26914	2.32837	0.00024	0.00344	0.02451	-0.00216
0.15000	-0.00577	-0.06638	-0.24620	2.26035	0.00028	0.00369	0.02448	-0.00232
0.20000	-0.00935	-0.07601	-0.14139	1.93609	0.00049	0.00491	0.02435	-0.00312
0.25000	-0.01329	-0.08078	-0.05216	1.63770	0.00077	0.00612	0.02417	-0.00395
0.30000	-0.01736	-0.08146	0.02280	1.36461	0.00110	0.00733	0.02395	-0.00480
0.35000	-0.02138	-0.07872	0.08472	1.11628	0.00150	0.00852	0.02369	-0.00568
0.40000	-0.02518	-0.07318	0.13483	0.89220	0.00196	0.00970	0.02338	-0.00661
0.45000	-0.02866	-0.06541	0.17433	0.69177	0.00247	0.01086	0.02303	-0.00757
0.50000	-0.03170	-0.05591	0.20439	0.51433	0.00304	0.01200	0.02263	-0.00857
0.55000	-0.03423	-0.04511	0.22614	0.35906	0.00367	0.01312	0.02217	-0.00962
0.60000	-0.03619	-0.03342	0.24065	0.22501	0.00435	0.01421	0.02166	-0.01071
0.65000	-0.03756	-0.02115	0.24897	0.11108	0.00509	0.01528	0.02110	-0.01185
0.70000	-0.03830	-0.00860	0.25207	0.01602	0.00588	0.01632	0.02048	-0.01303
0.80000	-0.03791	0.01642	0.24618	-0.12321	0.00761	0.01830	0.01905	-0.01549
0.90000	-0.03506	0.04027	0.22934	-0.20507	0.00954	0.02012	0.01738	-0.01806
1.00000	-0.02992	0.06210	0.20664	-0.24284	0.01163	0.02177	0.01544	-0.02064
1.10000	-0.02272	0.08153	0.18181	-0.24957	0.01388	0.02320	0.01325	-0.02313
1.20000	-0.01370	0.09847	0.15736	-0.23702	0.01626	0.02441	0.01082	-0.02540
1.40000	0.00884	0.12550	0.11448	-0.19010	0.02133	0.02604	0.00538	-0.02868
1.60000	0.03599	0.14489	0.08083	-0.14935	0.02661	0.02653	-0.00048	-0.02940
1.80000	0.06640	0.15824	0.05335	-0.12888	0.03186	0.02586	-0.00617	-0.02691
2.00000	0.09894	0.16638	0.02814	-0.12540	0.03688	0.02412	-0.01104	-0.02134
2.20000	0.13262	0.16947	0.00260	-0.13083	0.04146	0.02154	-0.01456	-0.01364
2.40000	0.16639	0.16733	-0.02420	-0.13638	0.04546	0.01841	-0.01645	-0.00524
2.60000	0.19919	0.15977	-0.05136	-0.13305	0.04880	0.01507	-0.01671	0.00239
2.80000	0.22994	0.14693	-0.07634	-0.11376	0.05149	0.01182	-0.01561	0.00817
3.00000	0.25766	0.12961	-0.09569	-0.07708	0.05355	0.00888	-0.01360	0.01161
3.20000	0.28158	0.10924	-0.10639	-0.02882	0.05507	0.00641	-0.01112	0.01280
3.40000	0.30128	0.08772	-0.10715	0.02040	0.05615	0.00444	-0.00859	0.01224
3.60000	0.31672	0.06698	-0.09889	0.05993	0.05688	0.00295	-0.00630	0.01057
3.80000	0.32823	0.04859	-0.08427	0.08333	0.05736	0.00189	-0.00440	0.00843
4.00000	0.33637	0.03347	-0.06670	0.08975	0.05766	0.00117	-0.00293	0.00628
4.20000	0.34185	0.02190	-0.04926	0.08282	0.05784	0.00069	-0.00187	0.00440
4.40000	0.34535	0.01361	-0.03409	0.06814	0.05795	0.00040	-0.00114	0.00293
4.60000	0.34748	0.00804	-0.02216	0.05103	0.05801	0.00022	-0.00067	0.00185
4.80000	0.34871	0.00452	-0.01358	0.03522	0.05804	0.00012	-0.00038	0.00112
5.00000	0.34938	0.00242	-0.00786	0.02259	0.05806	0.00006	-0.00021	0.00064
5.20000	0.34974	0.00123	-0.00431	0.01354	0.05807	0.00003	-0.00011	0.00036
5.40000	0.34991	0.00060	-0.00224	0.00762	0.05807	0.00001	-0.00005	0.00019
5.60000	0.35000	0.00028	-0.00110	0.00404	0.05807	0.00001	-0.00003	0.00010
5.80000	0.35004	0.00012	-0.00052	0.00202	0.05807	0.00000	-0.00001	0.00005
6.00000	0.35005	0.00005	-0.00023	0.00096	0.05807	0.00000	-0.00001	0.00002
6.20000	0.35006	0.00002	-0.00010	0.00043	0.05808	0.00000	-0.00000	0.00001
6.40000	0.35006	0.00001	-0.00004	0.00018	0.05808	-0.00000	-0.00000	0.00000
6.60000	0.35006	0.00000	-0.00002	0.00007	0.05808	-0.00000	-0.00000	0.00000
6.80000	0.35006	0.00000	-0.00001	0.00003	0.05807	-0.00000	-0.00000	0.00000
7.00000	0.35006	0.00000	-0.00000	0.00001	0.05807	-0.00000	-0.00000	0.00000
7.20000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
7.40000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
7.60000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
7.80000	0.35006	-0.00000	-0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
8.00000	0.35006	-0.00000	-0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
8.20000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
8.40000	0.35006	-0.00000	-0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
8.60000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
8.80000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
9.00000	0.35006	-0.00000	-0.00000	0.00000	0.05807	-0.00000	-0.00000	0.00000
9.20000	0.35006	-0.00000	-0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
9.40000	0.35006	-0.00000	0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
9.60000	0.35006	-0.00000	0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
9.80000	0.35006	-0.00000	0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000
10.00000	0.35006	-0.00000	0.00000	-0.00000	0.05807	-0.00000	-0.00000	0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_0 = 0.75$ - Continued

η	${}_3F_3$ $(g_1^2\delta_1)$	${}_3F_3'$	${}_3F_3''$	${}_3F_3'''$	${}_4F_3$ $(g_1\delta_1^2)$	${}_4F_3'$	${}_4F_3''$	${}_4F_3'''$
0.	0.	0.	0.01698	-1.20436	0.	0.	-0.01152	-0.01538
0.01000	0.00000	0.00011	0.00516	-1.16038	-0.00000	-0.00012	-0.01168	-0.01710
0.02000	0.00000	0.00010	-0.00623	-1.11709	-0.00000	-0.00023	-0.01186	-0.01878
0.03000	0.00000	-0.00001	-0.01719	-1.07448	-0.00001	-0.00035	-0.01205	-0.02044
0.04000	0.00000	-0.00024	-0.02772	-1.03254	-0.00001	-0.00047	-0.01227	-0.02206
0.05000	-0.00000	-0.00057	-0.03784	-0.99126	-0.00001	-0.00060	-0.01249	-0.02366
0.06000	-0.00001	-0.00099	-0.04755	-0.95063	-0.00002	-0.00072	-0.01274	-0.02522
0.07000	-0.00002	-0.00152	-0.05686	-0.91066	-0.00003	-0.00085	-0.01300	-0.02676
0.08000	-0.00004	-0.00213	-0.06576	-0.87133	-0.00004	-0.00098	-0.01327	-0.02827
0.09000	-0.00007	-0.00283	-0.07428	-0.83265	-0.00005	-0.00112	-0.01356	-0.02974
0.10000	-0.00010	-0.00361	-0.08242	-0.79460	-0.00006	-0.00126	-0.01387	-0.03119
0.11000	-0.00014	-0.00448	-0.09018	-0.75718	-0.00007	-0.00140	-0.01419	-0.03261
0.12000	-0.00019	-0.00542	-0.09757	-0.72040	-0.00009	-0.00154	-0.01452	-0.03400
0.13000	-0.00025	-0.00643	-0.10459	-0.68424	-0.00010	-0.00169	-0.01487	-0.03536
0.14000	-0.00032	-0.00751	-0.11125	-0.64870	-0.00012	-0.00184	-0.01523	-0.03669
0.15000	-0.00040	-0.00865	-0.11756	-0.61378	-0.00014	-0.00199	-0.01560	-0.03798
0.20000	-0.00099	-0.01523	-0.14406	-0.44843	-0.00026	-0.00282	-0.01765	-0.04397
0.25000	-0.00194	-0.02292	-0.16266	-0.29831	-0.00043	-0.00376	-0.01998	-0.04908
0.30000	-0.00329	-0.03137	-0.17414	-0.16331	-0.00064	-0.00482	-0.02255	-0.05320
0.35000	-0.00508	-0.04023	-0.17924	-0.04335	-0.00091	-0.00602	-0.02529	-0.05620
0.40000	-0.00732	-0.04920	-0.17872	0.06169	-0.00124	-0.00735	-0.02814	-0.05796
0.45000	-0.01000	-0.05802	-0.17332	0.15193	-0.00165	-0.00883	-0.03106	-0.05835
0.50000	-0.01311	-0.06647	-0.16377	0.22762	-0.00213	-0.01046	-0.03396	-0.05727
0.55000	-0.01664	-0.07434	-0.15080	0.28913	-0.00270	-0.01223	-0.03676	-0.05462
0.60000	-0.02054	-0.08150	-0.13509	0.33694	-0.00335	-0.01413	-0.03939	-0.05035
0.65000	-0.02477	-0.08782	-0.11732	0.37172	-0.00411	-0.01616	-0.04177	-0.04444
0.70000	-0.02930	-0.09321	-0.09812	0.39427	-0.00497	-0.01830	-0.04381	-0.03690
0.80000	-0.03905	-0.10101	-0.05773	0.40674	-0.00703	-0.02284	-0.04656	-0.01725
0.90000	-0.04937	-0.10478	-0.01796	0.38365	-0.00955	-0.02754	-0.04709	0.00751
1.00000	-0.05988	-0.10473	0.01817	0.33573	-0.01253	-0.03217	-0.04495	0.03557
1.10000	-0.07020	-0.10133	0.04873	0.27404	-0.01597	-0.03644	-0.03994	0.06453
1.20000	-0.08005	-0.09520	0.07286	0.20876	-0.01980	-0.04006	-0.03210	0.09170
1.40000	-0.09739	-0.07726	0.10280	0.09721	-0.02832	-0.04435	-0.00947	0.13010
1.60000	-0.11069	-0.05527	0.11493	0.03188	-0.03720	-0.04354	0.01767	0.13500
1.80000	-0.11941	-0.03188	0.11794	0.00175	-0.04538	-0.03746	0.04209	0.10378
2.00000	-0.12343	-0.00841	0.11598	-0.02272	-0.05191	-0.02731	0.05752	0.04798
2.20000	-0.12284	0.01410	0.10785	-0.06146	-0.05618	-0.01526	0.06095	-0.01282
2.40000	-0.11796	0.03412	0.09059	-0.11165	-0.05804	-0.00368	0.05331	-0.06022
2.60000	-0.10949	0.04969	0.06370	-0.15422	-0.05780	0.00558	0.03846	-0.08399
2.80000	-0.09849	0.05918	0.03072	-0.17010	-0.05603	0.01156	0.02132	-0.08389
3.00000	-0.08626	0.06199	-0.00208	-0.15255	-0.05340	0.01424	0.00603	-0.06696
3.20000	-0.07410	0.05878	-0.02856	-0.10918	-0.05052	0.01426	-0.00499	-0.04280
3.40000	-0.06304	0.05124	-0.04507	-0.05558	-0.04781	0.01257	-0.01116	-0.01957
3.60000	-0.05375	0.04145	-0.05115	-0.00702	-0.04554	0.01007	-0.01321	-0.00207
3.80000	-0.04648	0.03134	-0.04886	0.02712	-0.04379	0.00747	-0.01246	0.00835
4.00000	-0.04115	0.02225	-0.04142	0.04453	-0.04253	0.00518	-0.01028	0.01265
4.20000	-0.03746	0.01490	-0.03198	0.04803	-0.04168	0.00339	-0.00768	0.01279
4.40000	-0.03506	0.00944	-0.02280	0.04268	-0.04114	0.00210	-0.00530	0.01076
4.60000	-0.03358	0.00567	-0.01516	0.03340	-0.04082	0.00123	-0.00342	0.00803
4.80000	-0.03270	0.00324	-0.00947	0.02372	-0.04063	0.00069	-0.00208	0.00547
5.00000	-0.03221	0.00177	-0.00557	0.01554	-0.04053	0.00037	-0.00120	0.00346
5.20000	-0.03195	0.00092	-0.00311	0.00949	-0.04047	0.00019	-0.00065	0.00205
5.40000	-0.03182	0.00046	-0.00165	0.00544	-0.04044	0.00009	-0.00034	0.00115
5.60000	-0.03176	0.00022	-0.00083	0.00294	-0.04043	0.00004	-0.00017	0.00061
5.80000	-0.03173	0.00010	-0.00040	0.00151	-0.04042	0.00002	-0.00008	0.00031
6.00000	-0.03171	0.00004	-0.00018	0.00073	-0.04042	0.00001	-0.00004	0.00015
6.20000	-0.03171	0.00002	-0.00008	0.00034	-0.04042	0.00000	-0.00002	0.00007
6.40000	-0.03170	0.00001	-0.00003	0.00015	-0.04042	0.00000	-0.00001	0.00003
6.60000	-0.03170	0.00000	-0.00001	0.00006	-0.04042	0.00000	-0.00000	0.00001
6.80000	-0.03170	0.00000	-0.00001	0.00003	-0.04042	0.00000	-0.00000	0.00001
7.00000	-0.03170	0.00000	-0.00000	0.00001	-0.04042	0.00000	-0.00000	0.00000
7.20000	-0.03170	0.00000	-0.00000	0.00000	-0.04042	0.00000	-0.00000	0.00000
7.40000	-0.03170	0.00000	-0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
7.60000	-0.03170	0.00000	-0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
7.80000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
8.00000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
8.20000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
8.40000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
8.60000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	0.00000
8.80000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	-0.00000
9.00000	-0.03170	0.00000	0.00000	0.00000	-0.04042	0.00000	0.00000	-0.00000
9.20000	-0.03170	0.00000	0.00000	-0.00000	-0.04042	0.00000	0.00000	-0.00000
9.40000	-0.03170	0.00000	0.00000	-0.00000	-0.04042	0.00000	0.00000	-0.00000
9.60000	-0.03170	0.00000	0.00000	-0.00000	-0.04042	0.00000	0.00000	-0.00000
9.80000	-0.03170	0.00000	0.00000	-0.00000	-0.04042	0.00000	0.00000	-0.00000
10.00000	-0.03170	0.00000	0.00000	-0.00000	-0.04042	0.00000	0.00000	-0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_5F_3$ $(g_1 g_2)$	${}_5F_3'$	${}_5F_3''$	${}_5F_3'''$	${}_6F_3$ $(g_1 \delta_2)$	${}_6F_3'$	${}_6F_3''$	${}_6F_3'''$
0.	0.	0.	0.90002	-4.67971	0.	0.	0.00494	0.42268
0.01000	0.00004	0.00877	0.85377	-4.57039	0.00000	0.00007	0.00909	0.40768
0.02000	0.00017	0.01708	0.80861	-4.46247	0.00000	0.00018	0.01310	0.39288
0.03000	0.00038	0.02494	0.76452	-4.35594	0.00000	0.00033	0.01695	0.37829
0.04000	0.00067	0.03237	0.72148	-4.25080	0.00001	0.00052	0.02066	0.36390
0.05000	0.00103	0.03938	0.67950	-4.14701	0.00001	0.00074	0.02423	0.34971
0.06000	0.00146	0.04597	0.63854	-4.04459	0.00002	0.00100	0.02766	0.33572
0.07000	0.00195	0.05215	0.59860	-3.94350	0.00003	0.00130	0.03094	0.32192
0.08000	0.00250	0.05794	0.55966	-3.84374	0.00005	0.00162	0.03410	0.30832
0.09000	0.00311	0.06335	0.52172	-3.74529	0.00007	0.00198	0.03711	0.29491
0.10000	0.00376	0.06838	0.48475	-3.64816	0.00009	0.00236	0.03999	0.28169
0.11000	0.00447	0.07305	0.44875	-3.55232	0.00011	0.00278	0.04275	0.26866
0.12000	0.00522	0.07736	0.41370	-3.45777	0.00014	0.00322	0.04537	0.25581
0.13000	0.00602	0.08132	0.37959	-3.36450	0.00018	0.00368	0.04786	0.24316
0.14000	0.00685	0.08495	0.34641	-3.27250	0.00022	0.00418	0.05023	0.23069
0.15000	0.00772	0.08825	0.31414	-3.18177	0.00026	0.00469	0.05248	0.21840
0.20000	0.01246	0.10017	0.16605	-2.74689	0.00057	0.00756	0.06191	0.15968
0.25000	0.01762	0.10521	0.03894	-2.34283	0.00103	0.01083	0.06852	0.10546
0.30000	0.02288	0.10439	-0.06874	-1.96912	0.00165	0.01437	0.07253	0.05567
0.35000	0.02798	0.09863	-0.15848	-1.62540	0.00246	0.01805	0.07416	0.01030
0.40000	0.03268	0.08881	-0.23177	-1.31134	0.00346	0.02175	0.07363	-0.03065
0.45000	0.03680	0.07571	-0.29010	-1.02660	0.00464	0.02538	0.07117	-0.06715
0.50000	0.04020	0.06003	-0.33491	-0.77073	0.00599	0.02884	0.06699	-0.09920
0.55000	0.04277	0.04242	-0.36764	-0.54317	0.00752	0.03205	0.06132	-0.12678
0.60000	0.04442	0.02344	-0.38969	-0.34318	0.00919	0.03495	0.05439	-0.14991
0.65000	0.04510	0.00360	-0.40240	-0.16981	0.01101	0.03747	0.04641	-0.16861
0.70000	0.04478	-0.01666	-0.40709	-0.02192	0.01294	0.03958	0.03760	-0.18294
0.80000	0.04108	-0.05707	-0.39729	0.20307	0.01705	0.04239	0.01837	-0.19892
0.90000	0.03343	-0.09552	-0.36925	0.34505	0.02135	0.04322	-0.00166	-0.19908
1.00000	0.02209	-0.13057	-0.33052	0.41974	0.02563	0.04208	-0.02098	-0.18529
1.10000	0.00745	-0.16146	-0.28698	0.44412	0.02970	0.03909	-0.03833	-0.16002
1.20000	-0.01005	-0.18794	-0.24281	0.43496	0.03339	0.03451	-0.05270	-0.12622
1.40000	-0.05193	-0.22817	-0.16153	0.37369	0.03910	0.02197	-0.07006	-0.04599
1.60000	-0.10032	-0.25342	-0.09278	0.31875	0.04206	0.00757	-0.07137	0.03039
1.80000	-0.15245	-0.26580	-0.03180	0.29590	0.04221	-0.00569	-0.05943	0.08418
2.00000	-0.20585	-0.26629	0.02662	0.28818	0.04000	-0.01569	-0.03977	0.10728
2.20000	-0.25820	-0.25531	0.08245	0.26552	0.03621	-0.02149	-0.01842	0.10218
2.40000	-0.30727	-0.23382	0.13057	0.20975	0.03168	-0.02326	-0.00015	0.07836
2.60000	-0.35117	-0.20405	0.16431	0.12391	0.02712	-0.02193	0.01245	0.04733
2.80000	-0.38856	-0.16936	0.17936	0.02663	0.02303	-0.01869	0.01894	0.01852
3.00000	-0.41884	-0.13356	0.17570	-0.05982	0.01969	-0.01469	0.02038	-0.00269
3.20000	-0.44214	-0.10007	0.15724	-0.11963	0.01715	-0.01076	0.01847	-0.01491
3.40000	-0.45918	-0.07125	0.12999	-0.14761	0.01535	-0.00741	0.01493	-0.01945
3.60000	-0.47103	-0.04825	0.10005	-0.14782	0.01414	-0.00481	0.01104	-0.01883
3.80000	-0.47887	-0.03109	0.07211	-0.12941	0.01337	-0.00297	0.00757	-0.01555
4.00000	-0.48381	-0.01909	0.04886	-0.10228	0.01291	-0.00174	0.00486	-0.01151
4.20000	-0.48678	-0.01117	0.03124	-0.07426	0.01265	-0.00097	0.00294	-0.00781
4.40000	-0.48848	-0.00624	0.01890	-0.05005	0.01250	-0.00051	0.00168	-0.00493
4.60000	-0.48941	-0.00332	0.01084	-0.03154	0.01243	-0.00026	0.00091	-0.00291
4.80000	-0.48989	-0.00169	0.00590	-0.01868	0.01239	-0.00013	0.00047	-0.00162
5.00000	-0.49014	-0.00082	0.00306	-0.01043	0.01237	-0.00006	0.00023	-0.00085
5.20000	-0.49025	-0.00038	0.00151	-0.00551	0.01236	-0.00003	0.00011	-0.00042
5.40000	-0.49030	-0.00017	0.00071	-0.00276	0.01236	-0.00001	0.00005	-0.00020
5.60000	-0.49033	-0.00007	0.00032	-0.00131	0.01236	-0.00000	0.00002	-0.00009
5.80000	-0.49034	-0.00003	0.00014	-0.00059	0.01236	-0.00000	0.00001	-0.00004
6.00000	-0.49034	-0.00001	0.00006	-0.00026	0.01236	-0.00000	0.00000	-0.00002
6.20000	-0.49034	-0.00000	0.00002	-0.00010	0.01236	-0.00000	0.00000	-0.00001
6.40000	-0.49034	-0.00000	0.00001	-0.00004	0.01236	-0.00000	0.00000	-0.00000
6.60000	-0.49034	-0.00000	0.00000	-0.00002	0.01236	-0.00000	0.00000	-0.00000
6.80000	-0.49034	0.00000	0.00000	-0.00001	0.01236	-0.00000	-0.00000	-0.00000
7.00000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
7.20000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
7.40000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
7.60000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
7.80000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
8.00000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
8.20000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
8.40000	-0.49034	0.00000	0.00000	-0.00000	0.01236	-0.00000	-0.00000	-0.00000
8.60000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
8.80000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
9.00000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
9.20000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
9.40000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
9.60000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
9.80000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000
10.00000	-0.49034	0.00000	0.00000	0.00000	0.01236	-0.00000	-0.00000	0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_7F_3$ $(g_2\delta_1)$	$_7F_3'$	$_7F_3''$	$_7F_3'''$	$_8F_3$ $(\delta_1\delta_2)$	$_8F_3'$	$_8F_3''$	$_8F_3'''$
0.	0.	0.	-0.03577	1.19271	0.	0.	-0.08657	0.
0.01000	-0.00000	-0.00030	-0.02403	1.15513	-0.00000	-0.00087	-0.08656	0.00139
0.02000	-0.00001	-0.00048	-0.01267	1.11800	-0.00002	-0.00173	-0.08654	0.00276
0.03000	-0.00001	-0.00055	-0.00167	1.08130	-0.00004	-0.00260	-0.08651	0.00413
0.04000	-0.00002	-0.00052	0.00896	1.04505	-0.00007	-0.00346	-0.08646	0.00549
0.05000	-0.00002	-0.00038	0.01923	1.00922	-0.00011	-0.00433	-0.08640	0.00684
0.06000	-0.00002	-0.00013	0.02915	0.97382	-0.00016	-0.00519	-0.08632	0.00819
0.07000	-0.00002	0.00021	0.03871	0.93884	-0.00021	-0.00605	-0.08623	0.00953
0.08000	-0.00002	0.00064	0.04793	0.90429	-0.00028	-0.00691	-0.08613	0.01086
0.09000	-0.00001	0.00116	0.05680	0.87015	-0.00035	-0.00777	-0.08602	0.01219
0.10000	0.00000	0.00177	0.06533	0.83642	-0.00043	-0.00863	-0.08589	0.01352
0.11000	0.00003	0.00247	0.07353	0.80310	-0.00052	-0.00949	-0.08575	0.01484
0.12000	0.00005	0.00324	0.08139	0.77020	-0.00062	-0.01035	-0.08559	0.01616
0.13000	0.00009	0.00410	0.08893	0.73770	-0.00073	-0.01120	-0.08542	0.01747
0.14000	0.00014	0.00502	0.09615	0.70560	-0.00085	-0.01206	-0.08524	0.01878
0.15000	0.00019	0.00602	0.10305	0.67391	-0.00097	-0.01291	-0.08505	0.02009
0.20000	0.00063	0.01195	0.13289	0.52148	-0.00172	-0.01713	-0.08388	0.02660
0.25000	0.00141	0.01918	0.15536	0.37908	-0.00268	-0.02129	-0.08239	0.03308
0.30000	0.00257	0.02737	0.17097	0.24675	-0.00385	-0.02537	-0.08057	0.03953
0.35000	0.00416	0.03617	0.18021	0.12464	-0.00522	-0.02934	-0.07843	0.04594
0.40000	0.00619	0.04529	0.18360	0.01294	-0.00678	-0.03320	-0.07598	0.05228
0.45000	0.00869	0.05445	0.18168	-0.08813	-0.00854	-0.03694	-0.07321	0.05852
0.50000	0.01163	0.06338	0.17497	-0.17835	-0.01047	-0.04052	-0.07013	0.06461
0.55000	0.01502	0.07187	0.16403	-0.25752	-0.01259	-0.04394	-0.06675	0.07050
0.60000	0.01881	0.07972	0.14940	-0.32551	-0.01487	-0.04719	-0.06308	0.07612
0.65000	0.02297	0.08676	0.13166	-0.38225	-0.01730	-0.05025	-0.05914	0.08142
0.70000	0.02747	0.09285	0.11137	-0.42775	-0.01989	-0.05310	-0.05495	0.08632
0.80000	0.03724	0.10173	0.06533	-0.48564	-0.02546	-0.05815	-0.04588	0.09468
0.90000	0.04765	0.10579	0.01564	-0.50154	-0.03148	-0.06225	-0.03609	0.10071
1.00000	0.05823	0.10487	-0.03371	-0.47971	-0.03787	-0.06535	-0.02583	0.10404
1.10000	0.06847	0.09918	-0.07924	-0.42617	-0.04452	-0.06741	-0.01538	0.10441
1.20000	0.07792	0.08924	-0.11814	-0.34837	-0.05132	-0.06843	-0.00505	0.10174
1.40000	0.09301	0.05989	-0.16881	-0.15335	-0.06498	-0.06748	0.01409	0.08784
1.60000	0.10147	0.02440	-0.17959	0.04050	-0.07808	-0.06305	0.02949	0.06508
1.80000	0.10287	-0.00964	-0.15603	0.18403	-0.09002	-0.05603	0.03983	0.03802
2.00000	0.09810	-0.03656	-0.11091	0.25461	-0.10039	-0.04748	0.04474	0.01157
2.20000	0.08891	-0.05354	-0.05885	0.25572	-0.10898	-0.03846	0.04477	-0.01028
2.40000	0.07735	-0.06046	-0.01188	0.20801	-0.11580	-0.02982	0.04109	-0.02531
2.60000	0.06528	-0.05912	0.02286	0.13769	-0.12098	-0.02217	0.03513	-0.03314
2.80000	0.05407	-0.05228	0.04321	0.06734	-0.12475	-0.01583	0.02824	-0.03481
3.00000	0.04455	-0.04270	0.05076	0.01118	-0.12740	-0.01086	0.02149	-0.03213
3.20000	0.03703	-0.03260	0.04901	-0.02527	-0.12919	-0.00718	0.01555	-0.02703
3.40000	0.03145	-0.02345	0.04190	-0.04309	-0.13034	-0.00457	0.01074	-0.02110
3.60000	0.02753	-0.01597	0.03271	-0.04686	-0.13107	-0.00280	0.00709	-0.01547
3.80000	0.02493	-0.01035	0.02372	-0.04208	-0.13151	-0.00166	0.00449	-0.01072
4.00000	0.02328	-0.00639	0.01613	-0.03349	-0.13176	-0.00095	0.00273	-0.00706
4.20000	0.02229	-0.00377	0.01036	-0.02435	-0.13191	-0.00053	0.00160	-0.00443
4.40000	0.02171	-0.00213	0.00631	-0.01644	-0.13198	-0.00028	0.00090	-0.00266
4.60000	0.02139	-0.00116	0.00366	-0.01040	-0.13203	-0.00015	0.00049	-0.00154
4.80000	0.02122	-0.00060	0.00202	-0.00622	-0.13205	-0.00007	0.00026	-0.00085
5.00000	0.02113	-0.00030	0.00107	-0.00352	-0.13206	-0.00004	0.00013	-0.00045
5.20000	0.02109	-0.00015	0.00054	-0.00190	-0.13206	-0.00002	0.00006	-0.00023
5.40000	0.02107	-0.00007	0.00026	-0.00098	-0.13206	-0.00001	0.00003	-0.00012
5.60000	0.02106	-0.00003	0.00012	-0.00048	-0.13207	-0.00000	0.00001	-0.00006
5.80000	0.02106	-0.00001	0.00006	-0.00023	-0.13207	-0.00000	0.00001	-0.00003
6.00000	0.02105	-0.00001	0.00002	-0.00010	-0.13207	-0.00000	0.00000	-0.00001
6.20000	0.02105	-0.00000	0.00001	-0.00005	-0.13207	0.00000	0.00000	-0.00001
6.40000	0.02105	-0.00000	0.00000	-0.00002	-0.13207	0.00000	0.00000	-0.00000
6.60000	0.02105	-0.00000	0.00000	-0.00001	-0.13207	0.00000	0.00000	-0.00000
6.80000	0.02105	-0.00000	0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
7.00000	0.02105	-0.00000	0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
7.20000	0.02105	-0.00000	0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
7.40000	0.02105	-0.00000	-0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
7.60000	0.02105	-0.00000	-0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
7.80000	0.02105	-0.00000	-0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
8.00000	0.02105	-0.00000	-0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
8.20000	0.02105	-0.00000	-0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
8.40000	0.02105	-0.00000	-0.00000	-0.00000	-0.13207	0.00000	0.00000	-0.00000
8.60000	0.02105	-0.00000	-0.00000	0.00000	-0.13207	0.00000	0.00000	-0.00000
8.80000	0.02105	-0.00000	-0.00000	0.00000	-0.13207	0.00000	0.00000	-0.00000
9.00000	0.02105	-0.00000	-0.00000	0.00000	-0.13207	0.00000	0.00000	-0.00000
9.20000	0.02105	-0.00000	-0.00000	0.00000	-0.13207	0.00000	0.00000	-0.00000
9.40000	0.02105	-0.00000	-0.00000	0.00000	-0.13207	0.00000	0.00000	-0.00000
9.60000	0.02105	-0.00000	-0.00000	0.00000	-0.13207	0.00000	0.00000	-0.00000
9.80000	0.02105	-0.00000	-0.00000	0.00000	-0.13206	0.00000	0.00000	-0.00000
10.00000	0.02105	-0.00000	-0.00000	0.00000	-0.13206	0.00000	0.00000	-0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_9F_3$ (g_3)	$_9F_3'$	$_9F_3''$	$_9F_3'''$	$_{10}F_3$ (δ_3)	$_{10}F_3'$	$_{10}F_3''$	$_{10}F_3'''$
0.	0.	0.	-0.24834	1.34454	0.	0.	0.20051	-0.50000
0.01000	-0.00001	-0.00242	-0.23503	1.31719	0.00001	0.00198	0.19555	-0.49279
0.02000	-0.00005	-0.00470	-0.22200	1.29011	0.00004	0.00391	0.19065	-0.48565
0.03000	-0.00011	-0.00686	-0.20923	1.26330	0.00009	0.00579	0.18583	-0.47857
0.04000	-0.00018	-0.00889	-0.19673	1.23675	0.00016	0.00763	0.18108	-0.47156
0.05000	-0.00028	-0.01079	-0.18449	1.21045	0.00024	0.00942	0.17640	-0.46462
0.06000	-0.00040	-0.01258	-0.17252	1.18442	0.00034	0.01116	0.17179	-0.45775
0.07000	-0.00053	-0.01424	-0.16080	1.15864	0.00046	0.01285	0.16725	-0.45094
0.08000	-0.00068	-0.01579	-0.14935	1.13311	0.00060	0.01450	0.16277	-0.44419
0.09000	-0.00085	-0.01723	-0.13814	1.10783	0.00075	0.01611	0.15836	-0.43751
0.10000	-0.00103	-0.01856	-0.12719	1.08280	0.00092	0.01767	0.15402	-0.43089
0.11000	-0.00122	-0.01978	-0.11648	1.05801	0.00111	0.01919	0.14974	-0.42434
0.12000	-0.00142	-0.02089	-0.10603	1.03347	0.00131	0.02066	0.14553	-0.41784
0.13000	-0.00164	-0.02190	-0.09581	1.00917	0.00152	0.02210	0.14139	-0.41141
0.14000	-0.00186	-0.02281	-0.08584	0.98511	0.00175	0.02349	0.13730	-0.40504
0.15000	-0.00209	-0.02362	-0.07611	0.96128	0.00199	0.02484	0.13329	-0.39873
0.20000	-0.00335	-0.02627	-0.03096	0.84570	0.00339	0.03102	0.11412	-0.36805
0.25000	-0.00469	-0.02681	0.00856	0.73593	0.00508	0.03628	0.09646	-0.33879
0.30000	-0.00600	-0.02550	0.04273	0.63190	0.00700	0.04069	0.08022	-0.31091
0.35000	-0.00721	-0.02262	0.07184	0.53359	0.00913	0.04433	0.06534	-0.28436
0.40000	-0.00824	-0.01840	0.09618	0.44101	0.01142	0.04725	0.05176	-0.25911
0.45000	-0.00903	-0.01307	0.11604	0.35422	0.01385	0.04952	0.03941	-0.23513
0.50000	-0.00953	-0.00686	0.13170	0.27325	0.01637	0.05121	0.02823	-0.21238
0.55000	-0.00971	0.00003	0.14346	0.19815	0.01896	0.05236	0.01815	-0.19086
0.60000	-0.00952	0.00742	0.15161	0.12899	0.02160	0.05304	0.00912	-0.17053
0.65000	-0.00896	0.01514	0.15646	0.06577	0.02426	0.05329	0.00108	-0.15138
0.70000	-0.00800	0.02302	0.15829	0.00852	0.02692	0.05317	-0.00603	-0.13339
0.80000	-0.00491	0.03872	0.15411	-0.08821	0.03218	0.05195	-0.01771	-0.10080
0.90000	-0.00029	0.05356	0.14142	-0.16177	0.03728	0.04973	-0.02634	-0.07259
1.00000	0.00574	0.06680	0.12249	-0.21334	0.04211	0.04677	-0.03236	-0.04855
1.10000	0.01300	0.07792	0.09943	-0.24465	0.04661	0.04333	-0.03618	-0.02843
1.20000	0.02125	0.08661	0.07416	-0.25799	0.05076	0.03960	-0.03817	-0.01196
1.40000	0.03971	0.09630	0.02336	-0.24173	0.05791	0.03190	-0.03803	0.01133
1.60000	0.05913	0.09646	-0.02004	-0.18800	0.06355	0.02462	-0.03434	0.02409
1.80000	0.07779	0.08914	-0.05084	-0.11912	0.06782	0.01828	-0.02891	0.02915
2.00000	0.09447	0.07705	-0.06785	-0.05249	0.07094	0.01308	-0.02302	0.02908
2.20000	0.10847	0.06281	-0.07272	0.00106	0.07313	0.00904	-0.01748	0.02602
2.40000	0.11959	0.04856	-0.06861	0.03702	0.07462	0.00604	-0.01270	0.02157
2.60000	0.12799	0.03573	-0.05907	0.05576	0.07560	0.00390	-0.00887	0.01683
2.80000	0.13403	0.02508	-0.04723	0.06062	0.07623	0.00243	-0.00595	0.01245
3.00000	0.13818	0.01683	-0.03544	0.05614	0.07661	0.00147	-0.00384	0.00878
3.20000	0.14091	0.01081	-0.02511	0.04670	0.07684	0.00085	-0.00238	0.00591
3.40000	0.14263	0.00665	-0.01686	0.03573	0.07697	0.00048	-0.00142	0.00380
3.60000	0.14366	0.00392	-0.01077	0.02547	0.07704	0.00026	-0.00082	0.00235
3.80000	0.14426	0.00222	-0.00655	0.01705	0.07708	0.00014	-0.00045	0.00139
4.00000	0.14460	0.00120	-0.00380	0.01078	0.07710	0.00007	-0.00024	0.00078
4.20000	0.14477	0.00063	-0.00211	0.00645	0.07711	0.00003	-0.00012	0.00043
4.40000	0.14486	0.00031	-0.00112	0.00367	0.07711	0.00002	-0.00006	0.00022
4.60000	0.14491	0.00015	-0.00057	0.00198	0.07711	0.00001	-0.00003	0.00011
4.80000	0.14493	0.00007	-0.00027	0.00102	0.07711	0.00000	-0.00001	0.00005
5.00000	0.14494	0.00003	-0.00013	0.00050	0.07712	0.00000	-0.00001	0.00002
5.20000	0.14494	0.00001	-0.00006	0.00024	0.07712	0.00000	-0.00000	0.00001
5.40000	0.14494	0.00001	-0.00002	0.00011	0.07712	0.00000	-0.00000	0.00000
5.60000	0.14495	0.00000	-0.00001	0.00005	0.07712	-0.00000	-0.00000	0.00000
5.80000	0.14495	0.00000	-0.00000	0.00002	0.07712	-0.00000	-0.00000	0.00000
6.00000	0.14495	0.00000	-0.00000	0.00001	0.07712	-0.00000	-0.00000	0.00000
6.20000	0.14495	0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
6.40000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
6.60000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
6.80000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
7.00000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
7.20000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
7.40000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	0.00000
7.60000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	-0.00000
7.80000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	-0.00000
8.00000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	-0.00000
8.20000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	-0.00000
8.40000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	0.00000
8.60000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
8.80000	0.14495	-0.00000	-0.00000	0.00000	0.07712	-0.00000	-0.00000	0.00000
9.00000	0.14495	-0.00000	-0.00000	-0.00000	0.07712	-0.00000	-0.00000	0.00000
9.20000	0.14495	-0.00000	-0.00000	0.00000	0.07711	-0.00000	-0.00000	0.00000
9.40000	0.14495	-0.00000	-0.00000	0.00000	0.07711	-0.00000	-0.00000	0.00000
9.60000	0.14495	-0.00000	-0.00000	0.00000	0.07711	-0.00000	-0.00000	0.00000
9.80000	0.14495	-0.00000	-0.00000	0.00000	0.07711	-0.00000	-0.00000	0.00000
10.00000	0.14495	-0.00000	-0.00000	0.00000	0.07711	-0.00000	-0.00000	0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_1F_4$ $(g_1\ g_3)$	${}_1F_4'$	${}_1F_4''$	${}_1F_4'''$	${}_2F_4$ $(\delta_1\ \delta_3)$	${}_2F_4'$	${}_2F_4''$	${}_2F_4'''$
0.	0.	0.	1.37895	-8.38499	0.	0.	-0.08763	0.
0.01000	0.00007	0.01337	1.29629	-8.14696	-0.00000	-0.00088	-0.08762	0.00242
0.02000	0.00026	0.02593	1.21599	-7.91317	-0.00002	-0.00175	-0.08758	0.00479
0.03000	0.00058	0.03770	1.13801	-7.68358	-0.00004	-0.00263	-0.08752	0.00713
0.04000	0.00102	0.04870	1.06231	-7.45811	-0.00007	-0.00350	-0.08744	0.00942
0.05000	0.00156	0.05896	0.98884	-7.23672	-0.00011	-0.00438	-0.08733	0.01169
0.06000	0.00219	0.06849	0.91756	-7.01935	-0.00016	-0.00525	-0.08721	0.01391
0.07000	0.00292	0.07731	0.84844	-6.80594	-0.00021	-0.00612	-0.08706	0.01610
0.08000	0.00374	0.08546	0.78143	-6.59645	-0.00028	-0.00699	-0.08688	0.01826
0.09000	0.00463	0.09295	0.71649	-6.39084	-0.00035	-0.00786	-0.08669	0.02038
0.10000	0.00559	0.09980	0.65360	-6.18904	-0.00044	-0.00872	-0.08648	0.02248
0.11000	0.00662	0.10603	0.59270	-5.99102	-0.00053	-0.00959	-0.08624	0.02454
0.12000	0.00771	0.11166	0.53377	-5.79673	-0.00063	-0.01045	-0.08599	0.02657
0.13000	0.00885	0.11671	0.47675	-5.60613	-0.00074	-0.01131	-0.08571	0.02856
0.14000	0.01004	0.12120	0.42163	-5.41917	-0.00086	-0.01216	-0.08541	0.03053
0.15000	0.01128	0.12515	0.36836	-5.23582	-0.00098	-0.01302	-0.08510	0.03248
0.20000	0.01789	0.13739	0.12853	-4.37181	-0.00174	-0.01723	-0.08324	0.04176
0.25000	0.02483	0.13869	-0.07023	-3.59238	-0.00270	-0.02133	-0.08093	0.05039
0.30000	0.03161	0.13098	-0.23205	-2.89334	-0.00387	-0.02531	-0.07821	0.05838
0.35000	0.03781	0.11603	-0.36084	-2.27075	-0.00523	-0.02915	-0.07511	0.06575
0.40000	0.04312	0.09539	-0.46033	-1.72079	-0.00678	-0.03282	-0.07165	0.07250
0.45000	0.04728	0.07043	-0.53406	-1.23963	-0.00851	-0.03631	-0.06787	0.07862
0.50000	0.05011	0.04235	-0.58538	-0.82337	-0.01041	-0.03960	-0.06380	0.08408
0.55000	0.05148	0.01221	-0.61742	-0.46797	-0.01247	-0.04268	-0.05947	0.08888
0.60000	0.05131	-0.01912	-0.63312	-0.16922	-0.01467	-0.04554	-0.05492	0.09298
0.65000	0.04956	-0.05087	-0.63521	0.07724	-0.01702	-0.04817	-0.05018	0.09638
0.70000	0.04623	-0.08245	-0.62619	0.27597	-0.01949	-0.05056	-0.04529	0.09906
0.80000	0.03491	-0.14317	-0.58367	0.54892	-0.02475	-0.05459	-0.03521	0.10220
0.90000	0.01777	-0.19851	-0.52088	0.68753	-0.03037	-0.05759	-0.02495	0.10242
1.00000	-0.00457	-0.24706	-0.44940	0.72891	-0.03624	-0.05958	-0.01482	0.09980
1.10000	-0.03140	-0.28837	-0.37720	0.70722	-0.04225	-0.06057	-0.00508	0.09459
1.20000	-0.06201	-0.32264	-0.30908	0.65168	-0.04832	-0.06062	0.00402	0.08711
1.40000	-0.13189	-0.37230	-0.19189	0.52327	-0.06025	-0.05819	0.01954	0.06709
1.60000	-0.20952	-0.40086	-0.09633	0.44325	-0.07142	-0.05310	0.03063	0.04358
1.80000	-0.29104	-0.41149	-0.01080	0.41847	-0.08137	-0.04626	0.03699	0.02033
2.00000	-0.37300	-0.40534	0.07195	0.40712	-0.08987	-0.03860	0.03898	0.00031
2.20000	-0.45210	-0.38301	0.14994	0.36509	-0.09681	-0.03090	0.03745	-0.01470
2.40000	-0.52524	-0.34625	0.21463	0.27370	-0.10227	-0.02378	0.03347	-0.02412
2.60000	-0.58987	-0.29867	0.25682	0.14401	-0.10639	-0.01760	0.02815	-0.02833
2.80000	-0.64433	-0.24536	0.27161	0.00499	-0.10938	-0.01255	0.02242	-0.02837
3.00000	-0.68800	-0.19178	0.26024	-0.11305	-0.11148	-0.00861	0.01699	-0.02560
3.20000	-0.72134	-0.14259	0.22913	-0.19051	-0.11290	-0.00570	0.01228	-0.02131
3.40000	-0.74554	-0.10086	0.18709	-0.22266	-0.11382	-0.00364	0.00849	-0.01658
3.60000	-0.76227	-0.06791	0.14261	-0.21685	-0.11440	-0.00224	0.00563	-0.01217
3.80000	-0.77328	-0.04355	0.10198	-0.18658	-0.11475	-0.00133	0.00358	-0.00846
4.00000	-0.78018	-0.02662	0.06868	-0.14571	-0.11495	-0.00077	0.00219	-0.00560
4.20000	-0.78432	-0.01552	0.04368	-0.10486	-0.11507	-0.00043	0.00129	-0.00354
4.40000	-0.78667	-0.00864	0.02631	-0.07020	-0.11513	-0.00023	0.00073	-0.00214
4.60000	-0.78796	-0.00459	0.01503	-0.04401	-0.11517	-0.00012	0.00040	-0.00124
4.80000	-0.78863	-0.00233	0.00816	-0.02595	-0.11518	-0.00006	0.00021	-0.00069
5.00000	-0.78896	-0.00113	0.00422	-0.01444	-0.11519	-0.00003	0.00011	-0.00037
5.20000	-0.78912	-0.00053	0.00208	-0.00761	-0.11520	-0.00001	0.00005	-0.00019
5.40000	-0.78919	-0.00023	0.00097	-0.00380	-0.11520	-0.00001	0.00003	-0.00010
5.60000	-0.78923	-0.00010	0.00044	-0.00180	-0.11520	-0.00000	0.00001	-0.00005
5.80000	-0.78924	-0.00004	0.00019	-0.00081	-0.11520	-0.00000	0.00001	-0.00002
6.00000	-0.78924	-0.00002	0.00008	-0.00035	-0.11520	-0.00000	0.00000	-0.00001
6.20000	-0.78925	-0.00001	0.00003	-0.00014	-0.11520	-0.00000	0.00000	-0.00000
6.40000	-0.78925	-0.00000	0.00001	-0.00006	-0.11520	-0.00000	0.00000	-0.00000
6.60000	-0.78925	-0.00000	0.00000	-0.00002	-0.11520	-0.00000	0.00000	-0.00000
6.80000	-0.78925	-0.00000	0.00000	-0.00001	-0.11520	-0.00000	0.00000	-0.00000
7.00000	-0.78925	-0.00000	0.00000	-0.00000	-0.11520	-0.00000	0.00000	-0.00000
7.20000	-0.78925	-0.00000	0.00000	-0.00000	-0.11520	-0.00000	0.00000	-0.00000
7.40000	-0.78925	-0.00000	-0.00000	-0.00000	-0.11520	-0.00000	0.00000	-0.00000
7.60000	-0.78925	-0.00000	-0.00000	-0.00000	-0.11520	-0.00000	0.00000	-0.00000
7.80000	-0.78925	-0.00000	-0.00000	-0.00000	-0.11520	-0.00000	0.00000	-0.00000
8.00000	-0.78925	-0.00000	-0.00000	-0.00000	-0.11520	-0.00000	0.00000	-0.00000
8.20000	-0.78925	-0.00000	-0.00000	-0.00000	-0.11520	0.00000	0.00000	-0.00000
8.40000	-0.78925	-0.00000	-0.00000	-0.00000	-0.11520	0.00000	0.00000	-0.00000
8.60000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
8.80000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
9.00000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
9.20000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
9.40000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
9.60000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
9.80000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000
10.00000	-0.78925	-0.00000	-0.00000	0.00000	-0.11520	0.00000	0.00000	-0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_1F_5$ $(g_2\ g_3)$	${}_1F_5'$	${}_1F_5''$	${}_1F_5'''$	${}_1F_6$ $(g_1\ g_2\ g_3)$	${}_1F_6'$	${}_1F_6''$	${}_1F_6'''$
0.	0.	0.	3.96078	-27.71792	0.	0.	-57.43956	443.19953
0.01000	0.00019	0.03824	3.68843	-26.75533	-0.00280	-0.55255	-53.10033	424.74121
0.02000	0.00076	0.07380	3.42560	-25.81559	-0.01091	-1.06261	-48.94286	406.84478
0.03000	0.00166	0.10678	3.17205	-24.89828	-0.02392	-1.53199	-44.96160	389.49677
0.04000	0.00288	0.13727	2.92756	-24.00299	-0.04142	-1.96241	-41.15114	372.68409
0.05000	0.00440	0.16536	2.69191	-23.12934	-0.06304	-2.35556	-37.50618	356.39396
0.06000	0.00618	0.19114	2.46490	-22.27695	-0.08841	-2.71307	-34.02156	340.61405
0.07000	0.00821	0.21469	2.24631	-21.44545	-0.11719	-3.03651	-30.69224	325.33220
0.08000	0.01047	0.23609	2.03592	-20.63446	-0.14903	-3.32742	-27.51329	310.53666
0.09000	0.01293	0.25543	1.83355	-19.84365	-0.18363	-3.58726	-24.47992	296.21587
0.10000	0.01557	0.27279	1.63898	-19.07266	-0.22068	-3.81749	-21.58743	282.35862
0.11000	0.01838	0.28824	1.45203	-18.32116	-0.25989	-4.01947	-18.83124	268.95392
0.12000	0.02133	0.30185	1.27250	-17.58881	-0.30098	-4.19455	-16.20688	255.99100
0.13000	0.02441	0.31371	1.10019	-16.87530	-0.34369	-4.34403	-13.70998	243.45940
0.14000	0.02760	0.32388	0.93493	-16.18030	-0.38778	-4.46916	-11.33629	231.34882
0.15000	0.03088	0.33243	0.77653	-15.50352	-0.43300	-4.57115	-9.08164	219.64923
0.20000	0.04817	0.35323	0.08117	-12.38223	-0.66862	-4.77361	0.54473	166.97811
0.25000	0.06569	0.34298	-0.46856	-9.67259	-0.90338	-4.55677	7.76406	123.18381
0.30000	0.08206	0.30847	-0.89239	-7.34097	-1.11915	-4.03034	12.99337	87.20842
0.35000	0.09623	0.25553	-1.20842	-5.35562	-1.30277	-3.28446	16.59900	58.08122
0.40000	0.10739	0.18914	-1.43321	-3.68613	-1.44516	-2.39215	18.90068	34.90747
0.45000	0.11499	0.11348	-1.58181	-2.30310	-1.54052	-1.41149	20.17517	16.86113
0.50000	0.11864	0.03201	-1.66782	-1.17789	-1.58560	-0.38778	20.65952	3.18060
0.55000	0.11814	-0.05246	-1.70343	-0.28254	-1.57916	0.64465	20.55430	-6.83323
0.60000	0.11338	-0.13768	-1.69945	0.41019	-1.52142	1.66063	20.02666	-13.81769
0.65000	0.10439	-0.22191	-1.66535	0.92680	-1.41367	2.64257	19.21340	-18.34817
0.70000	0.09123	-0.30385	-1.60928	1.29282	-1.25792	3.57905	18.22415	-20.93793
0.80000	0.05304	-0.45751	-1.45773	1.66936	-0.81249	5.29306	16.03863	-22.03860
0.90000	0.00028	-0.59475	-1.28632	1.71745	-0.20660	6.78927	13.91994	-20.00346
1.00000	-0.06534	-0.71496	-1.12018	1.58635	0.53872	8.08637	12.07568	-16.81000
1.10000	-0.14218	-0.81937	-0.97113	1.39190	1.40507	9.21532	10.55591	-13.65735
1.20000	-0.22875	-0.90983	-0.84119	1.21440	2.37722	10.20714	9.32260	-11.13930
1.40000	-0.42600	-1.05531	-0.61854	1.06477	4.59144	11.87165	7.41051	-8.48261
1.60000	-0.64800	-1.15723	-0.39641	1.19009	7.10294	13.18894	5.77121	-8.19642
1.80000	-0.88571	-1.21127	-0.13689	1.39915	9.84500	14.17437	4.05125	-9.14270
2.00000	-1.12879	-1.20981	0.15420	1.47654	12.74823	14.79215	2.07507	-10.67681
2.20000	-1.36573	-1.15010	0.43738	1.31338	15.73338	14.98268	-0.22249	-12.24515
2.40000	-1.58536	-1.03861	0.66477	0.93133	18.70875	14.68595	-2.77209	-13.04876
2.60000	-1.77871	-0.89024	0.80238	0.43698	21.57317	13.87259	-5.33610	-12.28103
2.80000	-1.94029	-0.72434	0.84056	-0.04343	24.22530	12.57446	-7.55600	-9.61173
3.00000	-2.06854	-0.55980	0.79245	-0.41416	26.57753	10.89698	-9.07889	-5.43343
3.20000	-2.16529	-0.41131	0.68523	-0.63165	28.56968	9.00407	-9.69155	-0.69184
3.40000	-2.23473	-0.28758	0.54973	-0.70107	30.17724	7.08175	-9.39003	3.54429
3.60000	-2.28218	-0.19152	0.41228	-0.65868	31.41165	5.29683	-8.36064	6.49584
3.80000	-2.31308	-0.12160	0.29057	-0.55143	32.31308	3.76642	-6.89748	7.87644
4.00000	-2.33229	-0.07366	0.19319	-0.42142	32.93904	2.54636	-5.30407	7.85606
4.20000	-2.34367	-0.04260	0.12153	-0.29802	33.35245	1.63733	-3.81916	6.87578
4.40000	-2.35012	-0.02354	0.07249	-0.19669	33.61225	1.00182	-2.58416	5.43600
4.60000	-2.35362	-0.01243	0.04107	-0.12184	33.76767	0.58361	-1.64783	3.94471
4.80000	-2.35542	-0.00628	0.02214	-0.07113	33.85624	0.32388	-0.99259	2.65370
5.00000	-2.35632	-0.00303	0.01136	-0.03926	33.90434	0.17132	-0.56589	1.66614
5.20000	-2.35674	-0.00140	0.00556	-0.02053	33.92925	0.08643	-0.30585	0.98105
5.40000	-2.35693	-0.00062	0.00260	-0.01019	33.94156	0.04160	-0.15693	0.54369
5.60000	-2.35702	-0.00026	0.00116	-0.00481	33.94736	0.01912	-0.07652	0.28438
5.80000	-2.35705	-0.00011	0.00049	-0.00216	33.94998	0.00839	-0.03550	0.14070
6.00000	-2.35707	-0.00004	0.00020	-0.00092	33.95111	0.00352	-0.01568	0.06597
6.20000	-2.35707	-0.00002	0.00008	-0.00038	33.95157	0.00142	-0.00660	0.02935
6.40000	-2.35707	-0.00001	0.00003	-0.00015	33.95175	0.00055	-0.00265	0.01241
6.60000	-2.35708	-0.00000	0.00001	-0.00005	33.95182	0.00020	-0.00101	0.00499
6.80000	-2.35708	-0.00000	0.00000	-0.00002	33.95185	0.00008	-0.00037	0.00191
7.00000	-2.35708	-0.00000	0.00000	-0.00001	33.95186	0.00003	-0.00013	0.00070
7.20000	-2.35708	-0.00000	0.00000	-0.00000	33.95186	0.00001	-0.00004	0.00024
7.40000	-2.35708	-0.00000	0.00000	-0.00000	33.95186	0.00001	-0.00001	0.00008
7.60000	-2.35708	-0.00000	-0.00000	-0.00000	33.95187	0.00001	-0.00000	0.00002
7.80000	-2.35708	-0.00000	-0.00000	-0.00000	33.95187	0.00001	0.00000	-0.00000
8.00000	-2.35708	-0.00000	-0.00000	-0.00000	33.95187	0.00001	0.00000	-0.00000
8.20000	-2.35708	-0.00000	-0.00000	-0.00000	33.95187	0.00001	0.00000	0.00000
8.40000	-2.35708	-0.00000	-0.00000	-0.00000	33.95187	0.00001	0.00000	-0.00000
8.60000	-2.35708	-0.00000	-0.00000	0.00000	33.95187	0.00001	0.00000	0.00000
8.80000	-2.35708	-0.00000	-0.00000	0.00000	33.95188	0.00001	0.00000	0.00001
9.00000	-2.35708	-0.00000	-0.00000	0.00000	33.95188	0.00001	0.00000	0.00001
9.20000	-2.35708	-0.00000	-0.00000	0.00000	33.95188	0.00001	0.00000	-0.00001
9.40000	-2.35708	-0.00000	-0.00000	0.00000	33.95188	0.00001	0.00001	-0.00004
9.60000	-2.35708	-0.00000	-0.00000	0.00000	33.95188	0.00001	0.00001	-0.00004
9.80000	-2.35708	-0.00000	-0.00000	0.00000	33.95189	0.00001	0.00000	-0.00001
10.00000	-2.35708	-0.00000	-0.00000	0.00000	33.95189	0.00001	0.00001	-0.00006

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_1G_1$	$_1G_1'$	$_1G_1''$	$_1I_1$	$_2G_1$	$_2G_1'$	$_2G_1''$	$_2I_1$
0.	1.00000	-0.48312	0.	0.	0.	-0.24681	0.	0.
0.01000	0.99517	-0.48312	0.00002	0.00998	-0.00247	-0.24681	0.00003	-0.00001
0.02000	0.99034	-0.48312	0.00007	0.01990	-0.00494	-0.24681	0.00012	-0.00005
0.03000	0.98551	-0.48312	0.00015	0.02978	-0.00740	-0.24681	0.00028	-0.00011
0.04000	0.98068	-0.48312	0.00027	0.03961	-0.00987	-0.24680	0.00049	-0.00020
0.05000	0.97584	-0.48311	0.00042	0.04940	-0.01234	-0.24680	0.00077	-0.00031
0.06000	0.97101	-0.48311	0.00060	0.05913	-0.01481	-0.24679	0.00110	-0.00044
0.07000	0.96618	-0.48310	0.00081	0.06882	-0.01728	-0.24678	0.00150	-0.00060
0.08000	0.96135	-0.48309	0.00106	0.07845	-0.01974	-0.24676	0.00195	-0.00079
0.09000	0.95652	-0.48308	0.00134	0.08804	-0.02221	-0.24674	0.00247	-0.00100
0.10000	0.95169	-0.48306	0.00165	0.09758	-0.02468	-0.24671	0.00304	-0.00123
0.11000	0.94686	-0.48305	0.00199	0.10708	-0.02715	-0.24667	0.00368	-0.00149
0.12000	0.94203	-0.48302	0.00237	0.11652	-0.02961	-0.24663	0.00437	-0.00178
0.13000	0.93720	-0.48300	0.00277	0.12592	-0.03208	-0.24659	0.00511	-0.00209
0.14000	0.93237	-0.48297	0.00321	0.13527	-0.03454	-0.24653	0.00592	-0.00242
0.15000	0.92754	-0.48293	0.00368	0.14457	-0.03701	-0.24647	0.00678	-0.00278
0.20000	0.90340	-0.48268	0.00648	0.19034	-0.04932	-0.24601	0.01193	-0.00493
0.25000	0.87927	-0.48227	0.01001	0.23491	-0.06160	-0.24525	0.01845	-0.00771
0.30000	0.85517	-0.48167	0.01427	0.27827	-0.07384	-0.24414	0.02626	-0.01109
0.35000	0.83111	-0.48084	0.01920	0.32042	-0.08601	-0.24261	0.03529	-0.01509
0.40000	0.80709	-0.47974	0.02478	0.36138	-0.09809	-0.24059	0.04548	-0.01969
0.45000	0.78314	-0.47835	0.03098	0.40113	-0.11006	-0.23804	0.05671	-0.02490
0.50000	0.75927	-0.47663	0.03773	0.43969	-0.12189	-0.23490	0.06891	-0.03070
0.55000	0.73548	-0.47457	0.04501	0.47706	-0.13354	-0.23114	0.08194	-0.03708
0.60000	0.71181	-0.47212	0.05276	0.51324	-0.14499	-0.22670	0.09569	-0.04405
0.65000	0.68828	-0.46928	0.06094	0.54825	-0.15620	-0.22156	0.11002	-0.05158
0.70000	0.66489	-0.46602	0.06947	0.58207	-0.16713	-0.21569	0.12479	-0.05966
0.80000	0.61867	-0.45819	0.08740	0.64625	-0.18803	-0.20170	0.15502	-0.07743
0.90000	0.57332	-0.44852	0.10606	0.70584	-0.20737	-0.18469	0.18508	-0.09722
1.00000	0.52903	-0.43697	0.12489	0.76094	-0.22487	-0.16474	0.21361	-0.11885
1.10000	0.48599	-0.42355	0.14338	0.81168	-0.24023	-0.14207	0.23926	-0.14212
1.20000	0.44438	-0.40832	0.16097	0.85819	-0.25320	-0.11703	0.26076	-0.16681
1.40000	0.36615	-0.37295	0.19150	0.93912	-0.27118	-0.06183	0.28708	-0.21944
1.60000	0.29555	-0.33232	0.21311	1.00516	-0.27776	-0.00399	0.28670	-0.27452
1.80000	0.23344	-0.28845	0.22374	1.05791	-0.27296	0.05100	0.25876	-0.32978
2.00000	0.18023	-0.24359	0.22290	1.09913	-0.25790	0.09796	0.20756	-0.38302
2.20000	0.13591	-0.19999	0.21158	1.13060	-0.23458	0.13302	0.14138	-0.43239
2.40000	0.10003	-0.15952	0.19196	1.15406	-0.20562	0.15419	0.07040	-0.47648
2.60000	0.07180	-0.12357	0.16688	1.17112	-0.17383	0.16152	0.00438	-0.51444
2.80000	0.05025	-0.09293	0.13929	1.18322	-0.14182	0.15678	-0.04926	-0.54599
3.00000	0.03426	-0.06784	0.11179	1.19159	-0.11173	0.14292	-0.08644	-0.57130
3.20000	0.02275	-0.04807	0.08640	1.19722	-0.08503	0.12335	-0.10649	-0.59091
3.40000	0.01471	-0.03306	0.06436	1.20092	-0.06255	0.10133	-0.11146	-0.60560
3.60000	0.00926	-0.02206	0.04626	1.20328	-0.04449	0.07952	-0.10506	-0.61623
3.80000	0.00567	-0.01429	0.03211	1.20475	-0.03060	0.05978	-0.09150	-0.62367
4.00000	0.00338	-0.00898	0.02153	1.20564	-0.02036	0.04314	-0.07468	-0.62871
4.20000	0.00196	-0.00548	0.01395	1.20616	-0.01312	0.02992	-0.05760	-0.63201
4.40000	0.00111	-0.00324	0.00875	1.20646	-0.00818	0.01998	-0.04222	-0.63411
4.60000	0.00061	-0.00186	0.00530	1.20662	-0.00493	0.01286	-0.02952	-0.63540
4.80000	0.00032	-0.00104	0.00311	1.20671	-0.00288	0.00798	-0.01975	-0.63616
5.00000	0.00017	-0.00056	0.00177	1.20676	-0.00163	0.00478	-0.01267	-0.63660
5.20000	0.00008	-0.00030	0.00097	1.20679	-0.00090	0.00276	-0.00780	-0.63685
5.40000	0.00004	-0.00015	0.00052	1.20680	-0.00048	0.00154	-0.00462	-0.63698
5.60000	0.00002	-0.00007	0.00027	1.20680	-0.00024	0.00083	-0.00264	-0.63705
5.80000	0.00001	-0.00004	0.00013	1.20681	-0.00012	0.00043	-0.00145	-0.63709
6.00000	0.00000	-0.00002	0.00006	1.20681	-0.00006	0.00022	-0.00077	-0.63711
6.20000	0.00000	-0.00001	0.00003	1.20681	-0.00003	0.00011	-0.00039	-0.63711
6.40000	0.00000	-0.00000	0.00001	1.20681	-0.00001	0.00005	-0.00019	-0.63712
6.60000	0.00000	-0.00000	0.00001	1.20681	-0.00001	0.00002	-0.00009	-0.63712
6.80000	0.00000	-0.00000	0.00000	1.20681	-0.00000	0.00001	-0.00004	-0.63712
7.00000	0.00000	-0.00000	0.00000	1.20681	-0.00000	0.00000	-0.00002	-0.63712
7.20000	0.00000	-0.00000	0.00000	1.20681	-0.00000	0.00000	-0.00001	-0.63712
7.40000	0.00000	-0.00000	0.00000	1.20681	-0.00000	0.00000	-0.00000	-0.63712
7.60000	0.00000	-0.00000	0.00000	1.20681	-0.00000	0.00000	-0.00000	-0.63712
7.80000	0.00000	-0.00000	0.00000	1.20681	-0.00000	0.00000	-0.00000	-0.63712
8.00000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
8.20000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
8.40000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
8.60000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
8.80000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
9.00000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
9.20000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
9.40000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
9.60000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
9.80000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712
10.00000	0.00000	-0.00000	0.00000	1.20681	0.00000	0.00000	-0.00000	-0.63712

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_1G_2$ (m_1)	${}_1G_2'$	${}_1G_2''$	${}_1I_2$	${}_2G_2$ (g_1)	${}_2G_2'$	${}_2G_2''$	${}_2I_2$
0.	0.	0.22399	-0.46681	0.	0.	-0.00101	0.	0.
0.01000	0.00222	0.21933	-0.46682	0.00001	-0.00001	-0.00101	-0.00000	-0.00000
0.02000	0.00439	0.21466	-0.46684	0.00004	-0.00002	-0.00101	-0.00002	-0.00000
0.03000	0.00651	0.20999	-0.46687	0.00010	-0.00003	-0.00101	-0.00004	-0.00000
0.04000	0.00859	0.20532	-0.46691	0.00017	-0.00004	-0.00101	-0.00007	-0.00000
0.05000	0.01062	0.20065	-0.46695	0.00027	-0.00005	-0.00101	-0.00011	-0.00000
0.06000	0.01260	0.19598	-0.46699	0.00039	-0.00006	-0.00101	-0.00015	-0.00000
0.07000	0.01454	0.19131	-0.46704	0.00052	-0.00007	-0.00101	-0.00021	-0.00000
0.08000	0.01643	0.18664	-0.46708	0.00068	-0.00008	-0.00101	-0.00027	-0.00000
0.09000	0.01827	0.18197	-0.46712	0.00085	-0.00009	-0.00102	-0.00033	-0.00000
0.10000	0.02006	0.17730	-0.46715	0.00104	-0.00010	-0.00102	-0.00041	-0.00001
0.11000	0.02181	0.17263	-0.46717	0.00125	-0.00011	-0.00102	-0.00049	-0.00001
0.12000	0.02352	0.16796	-0.46717	0.00148	-0.00012	-0.00103	-0.00057	-0.00001
0.13000	0.02517	0.16328	-0.46716	0.00172	-0.00013	-0.00104	-0.00066	-0.00001
0.14000	0.02678	0.15861	-0.46714	0.00198	-0.00014	-0.00104	-0.00076	-0.00001
0.15000	0.02835	0.15394	-0.46709	0.00226	-0.00015	-0.00105	-0.00086	-0.00001
0.20000	0.03546	0.13060	-0.46647	0.00386	-0.00021	-0.00111	-0.00142	-0.00002
0.25000	0.04141	0.10731	-0.46499	0.00578	-0.00026	-0.00120	-0.00207	-0.00003
0.30000	0.04619	0.08412	-0.46237	0.00798	-0.00033	-0.00132	-0.00277	-0.00005
0.35000	0.04982	0.06109	-0.45836	0.01038	-0.00040	-0.00147	-0.00349	-0.00006
0.40000	0.05231	0.03831	-0.45272	0.01294	-0.00047	-0.00166	-0.00419	-0.00009
0.45000	0.05366	0.01585	-0.44523	0.01560	-0.00056	-0.00189	-0.00486	-0.00011
0.50000	0.05390	-0.00618	-0.43570	0.01829	-0.00066	-0.00215	-0.00547	-0.00014
0.55000	0.05305	-0.02768	-0.42399	0.02097	-0.00078	-0.00244	-0.00599	-0.00018
0.60000	0.05114	-0.04854	-0.40996	0.02358	-0.00091	-0.00275	-0.00640	-0.00022
0.65000	0.04821	-0.06864	-0.39355	0.02606	-0.00105	-0.00307	-0.00670	-0.00027
0.70000	0.04429	-0.08785	-0.37470	0.02838	-0.00122	-0.00341	-0.00686	-0.00033
0.80000	0.03370	-0.12315	-0.32972	0.03231	-0.00159	-0.00410	-0.00676	-0.00047
0.90000	0.01983	-0.15349	-0.27554	0.03501	-0.00203	-0.00475	-0.00606	-0.00065
1.00000	0.00320	-0.17799	-0.21337	0.03618	-0.00254	-0.00529	-0.00479	-0.00088
1.10000	-0.01555	-0.19596	-0.14506	0.03558	-0.00309	-0.00569	-0.00304	-0.00116
1.20000	-0.03576	-0.20688	-0.07302	0.03302	-0.00367	-0.00589	-0.00093	-0.00149
1.40000	-0.07762	-0.20696	0.07084	0.02169	-0.00483	-0.00561	0.00373	-0.00235
1.60000	-0.11672	-0.17989	0.19473	0.00216	-0.00585	-0.00443	0.00786	-0.00342
1.80000	-0.14816	-0.13175	0.27882	-0.02449	-0.00656	-0.00258	0.01038	-0.00467
2.00000	-0.16863	-0.07175	0.31242	-0.05637	-0.00686	-0.00042	0.01082	-0.00601
2.20000	-0.17676	-0.01011	0.29635	-0.09111	-0.00674	0.00163	0.00936	-0.00738
2.40000	-0.17317	0.04419	0.24146	-0.12629	-0.00624	0.00324	0.00664	-0.00868
2.60000	-0.15999	0.08500	0.16447	-0.15974	-0.00548	0.00425	0.00348	-0.00986
2.80000	-0.14025	0.10968	0.08292	-0.18984	-0.00458	0.00465	0.00057	-0.01087
3.00000	-0.11716	0.11882	0.01094	-0.21561	-0.00366	0.00453	-0.00164	-0.01169
3.20000	-0.09357	0.11527	-0.04293	-0.23668	-0.00279	0.00405	-0.00300	-0.01233
3.40000	-0.07163	0.10304	-0.07597	-0.25316	-0.00205	0.00338	-0.00357	-0.01282
3.60000	-0.05266	0.08616	-0.08997	-0.26553	-0.00144	0.00266	-0.00354	-0.01316
3.80000	-0.03725	0.06803	-0.08940	-0.27446	-0.00098	0.00199	-0.00314	-0.01340
4.00000	-0.02537	0.05102	-0.07953	-0.28066	-0.00064	0.00142	-0.00256	-0.01356
4.20000	-0.01667	0.03651	-0.06519	-0.28482	-0.00041	0.00097	-0.00195	-0.01367
4.40000	-0.01057	0.02500	-0.04997	-0.28751	-0.00025	0.00063	-0.00140	-0.01373
4.60000	-0.00647	0.01643	-0.03616	-0.28918	-0.00015	0.00040	-0.00096	-0.01377
4.80000	-0.00383	0.01037	-0.02485	-0.29019	-0.00008	0.00024	-0.00063	-0.01379
5.00000	-0.00219	0.00630	-0.01629	-0.29078	-0.00005	0.00014	-0.00039	-0.01381
5.20000	-0.00121	0.00369	-0.01021	-0.29111	-0.00002	0.00008	-0.00024	-0.01381
5.40000	-0.00065	0.00208	-0.00614	-0.29129	-0.00001	0.00004	-0.00014	-0.01382
5.60000	-0.00034	0.00113	-0.00354	-0.29139	-0.00001	0.00002	-0.00008	-0.01382
5.80000	-0.00017	0.00060	-0.00197	-0.29144	-0.00000	0.00001	-0.00004	-0.01382
6.00000	-0.00008	0.00030	-0.00105	-0.29146	-0.00000	0.00001	-0.00002	-0.01382
6.20000	-0.00004	0.00015	-0.00054	-0.29147	-0.00000	0.00000	-0.00001	-0.01382
6.40000	-0.00002	0.00007	-0.00027	-0.29148	-0.00000	0.00000	-0.00000	-0.01382
6.60000	-0.00001	0.00003	-0.00013	-0.29148	-0.00000	0.00000	-0.00000	-0.01382
6.80000	-0.00000	0.00001	-0.00006	-0.29148	-0.00000	0.00000	-0.00000	-0.01382
7.00000	-0.00000	0.00001	-0.00003	-0.29148	-0.00000	0.00000	-0.00000	-0.01382
7.20000	-0.00000	0.00000	-0.00001	-0.29148	-0.00000	0.00000	-0.00000	-0.01382
7.40000	-0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
7.60000	-0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
7.80000	-0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
8.00000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
8.20000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
8.40000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
8.60000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
8.80000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
9.00000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
9.20000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
9.40000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
9.60000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
9.80000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382
10.00000	0.00000	0.00000	-0.00000	-0.29148	0.00000	0.00000	-0.00000	-0.01382

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_3G_2$ (δ_1)	${}_3G_2'$	${}_3G_2''$	${}_3I_2$	${}_1G_3$ (m_1^2)	${}_1G_3'$	${}_1G_3''$	${}_1I_3$
0.	0.	-0.07178	0.	0.	0.	-0.60861	2.04972	0.
0.01000	-0.00072	-0.07178	0.00001	-0.00000	-0.00598	-0.58822	2.02945	-0.00003
0.02000	-0.00144	-0.07178	0.00006	-0.00001	-0.01176	-0.56802	2.00920	-0.00012
0.03000	-0.00215	-0.07178	0.00013	-0.00003	-0.01735	-0.54803	1.98897	-0.00026
0.04000	-0.00287	-0.07178	0.00023	-0.00006	-0.02273	-0.52824	1.96875	-0.00047
0.05000	-0.00359	-0.07178	0.00036	-0.00009	-0.02791	-0.50866	1.94854	-0.00072
0.06000	-0.00431	-0.07177	0.00052	-0.00013	-0.03290	-0.48927	1.92831	-0.00102
0.07000	-0.00502	-0.07177	0.00071	-0.00018	-0.03770	-0.47009	1.90806	-0.00138
0.08000	-0.00574	-0.07176	0.00092	-0.00023	-0.04230	-0.45111	1.88779	-0.00178
0.09000	-0.00646	-0.07175	0.00116	-0.00029	-0.04672	-0.43234	1.86747	-0.00222
0.10000	-0.00718	-0.07174	0.00143	-0.00036	-0.05095	-0.41376	1.84712	-0.00271
0.11000	-0.00789	-0.07172	0.00172	-0.00043	-0.05500	-0.39539	1.82671	-0.00324
0.12000	-0.00861	-0.07170	0.00204	-0.00052	-0.05886	-0.37723	1.80624	-0.00381
0.13000	-0.00933	-0.07168	0.00238	-0.00061	-0.06254	-0.35927	1.78571	-0.00442
0.14000	-0.01005	-0.07165	0.00275	-0.00070	-0.06604	-0.34151	1.76511	-0.00506
0.15000	-0.01076	-0.07162	0.00314	-0.00081	-0.06937	-0.32397	1.74443	-0.00574
0.20000	-0.01434	-0.07141	0.00545	-0.00143	-0.08343	-0.23935	1.63971	-0.00957
0.25000	-0.01790	-0.07107	0.00831	-0.00224	-0.09340	-0.16004	1.53246	-0.01401
0.30000	-0.02144	-0.07057	0.01166	-0.00322	-0.09953	-0.08615	1.42243	-0.01885
0.35000	-0.02496	-0.06990	0.01546	-0.00438	-0.10210	-0.01784	1.30963	-0.02391
0.40000	-0.02843	-0.06902	0.01964	-0.00572	-0.10141	0.04477	1.19429	-0.02901
0.45000	-0.03185	-0.06793	0.02414	-0.00723	-0.09772	0.10155	1.07688	-0.03400
0.50000	-0.03522	-0.06660	0.02890	-0.00890	-0.09135	0.15243	0.95807	-0.03873
0.55000	-0.03851	-0.06504	0.03385	-0.01075	-0.08258	0.19735	0.83869	-0.04309
0.60000	-0.04172	-0.06322	0.03893	-0.01275	-0.07171	0.23631	0.71973	-0.04696
0.65000	-0.04483	-0.06114	0.04407	-0.01492	-0.05905	0.26935	0.60230	-0.05023
0.70000	-0.04783	-0.05881	0.04920	-0.01723	-0.04488	0.29658	0.48760	-0.05284
0.80000	-0.05345	-0.05339	0.05915	-0.02230	-0.01315	0.33435	0.27126	-0.05577
0.90000	-0.05847	-0.04701	0.06825	-0.02790	0.02131	0.35168	0.08028	-0.05538
1.00000	-0.06282	-0.03978	0.07601	-0.03397	0.05660	0.35153	-0.07709	-0.05148
1.10000	-0.06641	-0.03186	0.08201	-0.04044	0.09116	0.33758	-0.19508	-0.04408
1.20000	-0.06918	-0.02345	0.08593	-0.04723	0.12380	0.31391	-0.27131	-0.03331
1.40000	-0.07212	-0.00602	0.08677	-0.06142	0.18067	0.25365	-0.30778	-0.00266
1.60000	-0.07163	0.01064	0.07837	-0.07585	0.22560	0.19798	-0.23760	0.03815
1.80000	-0.06804	0.02482	0.06241	-0.08986	0.26110	0.16031	-0.14158	0.08695
2.00000	-0.06196	0.03530	0.04191	-0.10290	0.29078	0.13825	-0.09057	0.14221
2.20000	-0.05420	0.04151	0.02033	-0.11453	0.31659	0.11897	-0.11455	0.20301
2.40000	-0.04563	0.04357	0.00079	-0.12452	0.33761	0.08850	-0.19646	0.26853
2.60000	-0.03701	0.04211	-0.01455	-0.13278	0.35073	0.03960	-0.29017	0.33753
2.80000	-0.02896	0.03810	-0.02470	-0.13937	0.35237	-0.02514	-0.34822	0.40805
3.00000	-0.02187	0.03257	-0.02974	-0.14443	0.34030	-0.09544	-0.34355	0.47756
3.20000	-0.01596	0.02649	-0.03052	-0.14819	0.31470	-0.15836	-0.27649	0.54326
3.40000	-0.01127	0.02057	-0.02824	-0.15090	0.27817	-0.20329	-0.16843	0.60270
3.60000	-0.00769	0.01531	-0.02419	-0.15277	0.23494	-0.22498	-0.04904	0.65408
3.80000	-0.00508	0.01094	-0.01945	-0.15404	0.18970	-0.22392	0.05551	0.69655
4.00000	-0.00325	0.00752	-0.01480	-0.15486	0.14658	-0.20485	0.12931	0.73011
4.20000	-0.00202	0.00498	-0.01072	-0.15538	0.10851	-0.17456	0.16789	0.75552
4.40000	-0.00121	0.00318	-0.00741	-0.15569	0.07705	-0.13977	0.17556	0.77396
4.60000	-0.00070	0.00196	-0.00491	-0.15588	0.05254	-0.10580	0.16126	0.78680
4.80000	-0.00040	0.00117	-0.00313	-0.15599	0.03444	-0.07607	0.13485	0.79540
5.00000	-0.00022	0.00067	-0.00191	-0.15605	0.02173	-0.05212	0.10456	0.80094
5.20000	-0.00012	0.00037	-0.00113	-0.15608	0.01320	-0.03412	0.07602	0.80437
5.40000	-0.00006	0.00020	-0.00064	-0.15610	0.00773	-0.02139	0.05220	0.80642
5.60000	-0.00003	0.00010	-0.00035	-0.15611	0.00436	-0.01286	0.03402	0.80760
5.80000	-0.00001	0.00005	-0.00019	-0.15611	0.00238	-0.00743	0.02112	0.80826
6.00000	-0.00001	0.00003	-0.00009	-0.15611	0.00125	-0.00412	0.01253	0.80861
6.20000	-0.00000	0.00001	-0.00005	-0.15611	0.00063	-0.00220	0.00711	0.80879
6.40000	-0.00000	0.00001	-0.00002	-0.15611	0.00031	-0.00113	0.00387	0.80888
6.60000	-0.00000	0.00000	-0.00001	-0.15611	0.00015	-0.00056	0.00202	0.80893
6.80000	-0.00000	0.00000	-0.00000	-0.15611	0.00007	-0.00027	0.00102	0.80895
7.00000	-0.00000	0.00000	-0.00000	-0.15611	0.00003	-0.00012	0.00049	0.80896
7.20000	-0.00000	0.00000	-0.00000	-0.15611	0.00001	-0.00006	0.00023	0.80896
7.40000	-0.00000	0.00000	-0.00000	-0.15611	0.00001	-0.00002	0.00010	0.80896
7.60000	-0.00000	0.00000	-0.00000	-0.15611	0.00000	-0.00001	0.00004	0.80896
7.80000	-0.00000	0.00000	-0.00000	-0.15611	0.00000	-0.00000	0.00002	0.80896
8.00000	-0.00000	0.00000	-0.00000	-0.15611	0.00000	-0.00000	0.00001	0.80896
8.20000	-0.00000	0.00000	-0.00000	-0.15611	0.00000	-0.00000	0.00000	0.80896
8.40000	-0.00000	0.00000	-0.00000	-0.15611	0.00000	-0.00000	0.00000	0.80896
8.60000	-0.00000	0.00000	-0.00000	-0.15611	0.00000	-0.00000	0.00000	0.80896
8.80000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896
9.00000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896
9.20000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896
9.40000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896
9.60000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896
9.80000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896
10.00000	-0.00000	0.00000	-0.00000	-0.15611	-0.00000	-0.00000	0.00000	0.80896

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_2G_3$ $(m_1 g_1)$	$_2G'_3$	$_2G''_3$	$_2I_3$	$_3G_3$ (g_1^2)	$_3G'_3$	$_3G''_3$	$_3I_3$
0.	0.	0.01685	-0.00292	0.	0.	0.00283	-0.	0.
0.01000	0.00017	0.01682	-0.00291	0.00000	0.00003	0.00283	0.00001	0.00000
0.02000	0.00034	0.01679	-0.00291	0.00000	0.00006	0.00283	0.00005	0.00000
0.03000	0.00050	0.01676	-0.00290	0.00001	0.00009	0.00283	0.00011	0.00000
0.04000	0.00067	0.01673	-0.00288	0.00001	0.00011	0.00284	0.00019	0.00000
0.05000	0.00084	0.01670	-0.00287	0.00002	0.00014	0.00284	0.00030	0.00000
0.06000	0.00101	0.01667	-0.00286	0.00003	0.00017	0.00284	0.00042	0.00001
0.07000	0.00117	0.01664	-0.00285	0.00004	0.00020	0.00285	0.00056	0.00001
0.08000	0.00134	0.01662	-0.00284	0.00005	0.00023	0.00285	0.00073	0.00001
0.09000	0.00150	0.01659	-0.00283	0.00007	0.00026	0.00286	0.00091	0.00001
0.10000	0.00167	0.01656	-0.00284	0.00008	0.00028	0.00287	0.00110	0.00001
0.11000	0.00184	0.01653	-0.00284	0.00010	0.00031	0.00288	0.00131	0.00002
0.12000	0.00200	0.01650	-0.00286	0.00012	0.00034	0.00290	0.00154	0.00002
0.13000	0.00217	0.01647	-0.00288	0.00014	0.00037	0.00291	0.00178	0.00002
0.14000	0.00233	0.01644	-0.00291	0.00016	0.00040	0.00293	0.00203	0.00003
0.15000	0.00249	0.01642	-0.00295	0.00019	0.00043	0.00295	0.00229	0.00003
0.20000	0.00331	0.01626	-0.00332	0.00033	0.00058	0.00311	0.00376	0.00006
0.25000	0.00412	0.01608	-0.00403	0.00052	0.00074	0.00333	0.00540	0.00009
0.30000	0.00492	0.01585	-0.00510	0.00074	0.00092	0.00365	0.00711	0.00013
0.35000	0.00570	0.01556	-0.00657	0.00101	0.00111	0.00404	0.00883	0.00018
0.40000	0.00647	0.01519	-0.00842	0.00132	0.00132	0.00453	0.01047	0.00024
0.45000	0.00722	0.01471	-0.01063	0.00166	0.00156	0.00509	0.01197	0.00031
0.50000	0.00794	0.01412	-0.01312	0.00204	0.00183	0.00572	0.01327	0.00040
0.55000	0.00863	0.01340	-0.01582	0.00245	0.00213	0.00641	0.01433	0.00050
0.60000	0.00928	0.01254	-0.01865	0.00290	0.00247	0.00715	0.01511	0.00061
0.65000	0.00988	0.01153	-0.02150	0.00338	0.00285	0.00792	0.01558	0.00075
0.70000	0.01043	0.01039	-0.02425	0.00389	0.00327	0.00870	0.01573	0.00090
0.80000	0.01134	0.00771	-0.02903	0.00498	0.00421	0.01025	0.01501	0.00127
0.90000	0.01196	0.00464	-0.03211	0.00614	0.00531	0.01166	0.01299	0.00175
1.00000	0.01226	0.00137	-0.03281	0.00736	0.00654	0.01281	0.00981	0.00234
1.10000	0.01224	-0.00183	-0.03076	0.00859	0.00786	0.01359	0.00572	0.00306
1.20000	0.01191	-0.00469	-0.02594	0.00980	0.00924	0.01393	0.00100	0.00391
1.40000	0.01055	-0.00838	-0.00984	0.01205	0.01198	0.01313	-0.00898	0.00604
1.60000	0.00880	-0.00843	0.00903	0.01399	0.01437	0.01043	-0.01762	0.00868
1.80000	0.00740	-0.00512	0.02274	0.01560	0.01606	0.00630	-0.02305	0.01174
2.00000	0.00687	-0.00004	0.02616	0.01701	0.01684	0.00148	-0.02443	0.01504
2.20000	0.00736	0.00465	0.01919	0.01841	0.01666	-0.00321	-0.02195	0.01841
2.40000	0.00859	0.00722	0.00603	0.02000	0.01561	-0.00711	-0.01661	0.02165
2.60000	0.01006	0.00704	-0.00753	0.02187	0.01390	-0.00976	-0.00985	0.02461
2.80000	0.01125	0.00450	-0.01693	0.02401	0.01180	-0.01105	-0.00313	0.02718
3.00000	0.01177	0.00068	-0.02020	0.02632	0.00956	-0.01109	0.00244	0.02932
3.20000	0.01151	-0.00322	-0.01797	0.02866	0.00742	-0.01019	0.00625	0.03102
3.40000	0.01054	-0.00628	-0.01232	0.03088	0.00553	-0.00872	0.00822	0.03231
3.60000	0.00909	-0.00807	-0.00562	0.03285	0.00395	-0.00701	0.00862	0.03325
3.80000	0.00740	-0.00858	0.00028	0.03450	0.00272	-0.00534	0.00795	0.03391
4.00000	0.00572	-0.00808	0.00443	0.03581	0.00180	-0.00387	0.00668	0.03436
4.20000	0.00421	-0.00694	0.00661	0.03680	0.00115	-0.00268	0.00521	0.03465
4.40000	0.00296	-0.00554	0.00716	0.03751	0.00071	-0.00178	0.00383	0.03483
4.60000	0.00199	-0.00416	0.00659	0.03800	0.00042	-0.00113	0.00266	0.03494
4.80000	0.00128	-0.00294	0.00546	0.03832	0.00025	-0.00069	0.00177	0.03501
5.00000	0.00080	-0.00198	0.00416	0.03853	0.00014	-0.00041	0.00112	0.03505
5.20000	0.00047	-0.00127	0.00296	0.03865	0.00007	-0.00023	0.00068	0.03507
5.40000	0.00027	-0.00078	0.00199	0.03873	0.00004	-0.00013	0.00040	0.03508
5.60000	0.00015	-0.00046	0.00127	0.03877	0.00002	-0.00007	0.00022	0.03508
5.80000	0.00008	-0.00026	0.00077	0.03879	0.00001	-0.00003	0.00012	0.03509
6.00000	0.00004	-0.00014	0.00044	0.03880	0.00000	-0.00002	0.00006	0.03509
6.20000	0.00002	-0.00007	0.00025	0.03881	0.00000	-0.00001	0.00003	0.03509
6.40000	0.00001	-0.00004	0.00013	0.03881	0.00000	-0.00000	0.00002	0.03509
6.60000	0.00000	-0.00002	0.00007	0.03881	0.00000	-0.00000	0.00001	0.03509
6.80000	0.00000	-0.00001	0.00003	0.03881	0.00000	-0.00000	0.00000	0.03509
7.00000	0.00000	-0.00000	0.00002	0.03881	0.00000	-0.00000	0.00000	0.03509
7.20000	0.00000	-0.00000	0.00001	0.03881	0.00000	-0.00000	0.00000	0.03509
7.40000	0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
7.60000	0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
7.80000	0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
8.00000	0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
8.20000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
8.40000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
8.60000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
8.80000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
9.00000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
9.20000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
9.40000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
9.60000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
9.80000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509
10.00000	-0.00000	-0.00000	0.00000	0.03881	0.00000	-0.00000	0.00000	0.03509

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_0 = 0.75$ - Continued

η	${}_4G_3$ $(g_1\delta_1)$	${}_4G_3'$	${}_4G_3''$	${}_4I_3$	${}_5G_3$ (δ_1^2)	${}_5G_3'$	${}_5G_3''$	${}_5I_3$
0.	0.	-0.02332	0.	0.	0.	0.03056	-0.	0.
0.01000	-0.00023	-0.02332	-0.00000	-0.00000	0.00031	0.03056	-0.00000	0.00000
0.02000	-0.00047	-0.02332	-0.00000	-0.00000	0.00061	0.03056	-0.00000	0.00001
0.03000	-0.00070	-0.02332	-0.00000	-0.00001	0.00092	0.03056	-0.00001	0.00001
0.04000	-0.00093	-0.02332	-0.00000	-0.00002	0.00122	0.03056	-0.00001	0.00002
0.05000	-0.00117	-0.02332	-0.00000	-0.00003	0.00153	0.03056	-0.00002	0.00004
0.06000	-0.00140	-0.02332	0.00000	-0.00004	0.00183	0.03056	-0.00002	0.00006
0.07000	-0.00163	-0.02332	0.00001	-0.00006	0.00214	0.03056	-0.00003	0.00007
0.08000	-0.00187	-0.02332	0.00001	-0.00007	0.00244	0.03056	-0.00004	0.00010
0.09000	-0.00210	-0.02332	0.00002	-0.00009	0.00275	0.03056	-0.00006	0.00012
0.10000	-0.00233	-0.02332	0.00003	-0.00012	0.00306	0.03056	-0.00007	0.00015
0.11000	-0.00257	-0.02332	0.00005	-0.00014	0.00336	0.03056	-0.00008	0.00018
0.12000	-0.00280	-0.02332	0.00007	-0.00017	0.00367	0.03056	-0.00010	0.00022
0.13000	-0.00303	-0.02332	0.00009	-0.00020	0.00397	0.03056	-0.00012	0.00026
0.14000	-0.00326	-0.02332	0.00012	-0.00023	0.00428	0.03055	-0.00014	0.00030
0.15000	-0.00350	-0.02332	0.00016	-0.00026	0.00458	0.03055	-0.00017	0.00034
0.20000	-0.00466	-0.02330	0.00041	-0.00047	0.00611	0.03054	-0.00032	0.00061
0.25000	-0.00583	-0.02327	0.00083	-0.00073	0.00764	0.03052	-0.00054	0.00095
0.30000	-0.00699	-0.02322	0.00145	-0.00105	0.00916	0.03049	-0.00083	0.00137
0.35000	-0.00815	-0.02312	0.00230	-0.00143	0.01069	0.03044	-0.00121	0.00187
0.40000	-0.00930	-0.02298	0.00338	-0.00186	0.01221	0.03036	-0.00168	0.00244
0.45000	-0.01045	-0.02278	0.00472	-0.00236	0.01372	0.03027	-0.00226	0.00309
0.50000	-0.01158	-0.02251	0.00629	-0.00291	0.01523	0.03014	-0.00295	0.00382
0.55000	-0.01270	-0.02215	0.00811	-0.00352	0.01674	0.02997	-0.00377	0.00461
0.60000	-0.01379	-0.02169	0.01016	-0.00418	0.01823	0.02976	-0.00472	0.00549
0.65000	-0.01486	-0.02113	0.01240	-0.00489	0.01971	0.02949	-0.00582	0.00644
0.70000	-0.01590	-0.02045	0.01480	-0.00566	0.02118	0.02917	-0.00706	0.00746
0.80000	-0.01786	-0.01871	0.01996	-0.00735	0.02405	0.02833	-0.00998	0.00972
0.90000	-0.01963	-0.01645	0.02524	-0.00923	0.02683	0.02716	-0.01346	0.01227
1.00000	-0.02114	-0.01368	0.03022	-0.01127	0.02947	0.02562	-0.01742	0.01508
1.10000	-0.02235	-0.01043	0.03445	-0.01345	0.03194	0.02366	-0.02172	0.01816
1.20000	-0.02321	-0.00682	0.03753	-0.01573	0.03419	0.02127	-0.02616	0.02146
1.40000	-0.02381	0.00095	0.03909	-0.02046	0.03786	0.01518	-0.03448	0.02869
1.60000	-0.02286	0.00836	0.03397	-0.02515	0.04017	0.00764	-0.04030	0.03652
1.80000	-0.02057	0.01417	0.02347	-0.02951	0.04087	-0.00066	-0.04192	0.04465
2.00000	-0.01735	0.01759	0.01050	-0.03331	0.03991	-0.00879	-0.03862	0.05275
2.20000	-0.01371	0.01843	-0.00172	-0.03642	0.03743	-0.01581	-0.03091	0.06051
2.40000	-0.01012	0.01711	-0.01081	-0.03880	0.03371	-0.02096	-0.02030	0.06764
2.60000	-0.00696	0.01438	-0.01577	-0.04050	0.02919	-0.02387	-0.00881	0.07394
2.80000	-0.00441	0.01106	-0.01691	-0.04162	0.02432	-0.02455	0.00168	0.07930
3.00000	-0.00253	0.00780	-0.01531	-0.04231	0.01950	-0.02335	0.00984	0.08367
3.20000	-0.00126	0.00503	-0.01226	-0.04268	0.01506	-0.02082	0.01505	0.08712
3.40000	-0.00047	0.00292	-0.00884	-0.04284	0.01122	-0.01753	0.01737	0.08974
3.60000	-0.00005	0.00147	-0.00576	-0.04289	0.00807	-0.01403	0.01732	0.09166
3.80000	0.00015	0.00057	-0.00336	-0.04288	0.00560	-0.01071	0.01566	0.09301
4.00000	0.00021	0.00008	-0.00170	-0.04284	0.00375	-0.00782	0.01311	0.09394
4.20000	0.00020	-0.00015	-0.00066	-0.04280	0.00243	-0.00548	0.01030	0.09455
4.40000	0.00016	-0.00022	-0.00009	-0.04276	0.00153	-0.00369	0.00766	0.09494
4.60000	0.00012	-0.00021	0.00016	-0.04273	0.00092	-0.00239	0.00541	0.09518
4.80000	0.00008	-0.00017	0.00024	-0.04271	0.00054	-0.00149	0.00365	0.09532
5.00000	0.00005	-0.00012	0.00022	-0.04270	0.00031	-0.00090	0.00236	0.09541
5.20000	0.00003	-0.00008	0.00018	-0.04269	0.00017	-0.00052	0.00146	0.09545
5.40000	0.00002	-0.00005	0.00012	-0.04269	0.00009	-0.00029	0.00087	0.09548
5.60000	0.00001	-0.00003	0.00008	-0.04269	0.00005	-0.00016	0.00050	0.09549
5.80000	0.00000	-0.00002	0.00005	-0.04269	0.00002	-0.00008	0.00027	0.09550
6.00000	0.00000	-0.00001	0.00003	-0.04268	0.00001	-0.00004	0.00015	0.09550
6.20000	0.00000	-0.00000	0.00002	-0.04268	0.00001	-0.00002	0.00007	0.09550
6.40000	0.00000	-0.00000	0.00001	-0.04268	0.00000	-0.00001	0.00004	0.09550
6.60000	0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00002	0.09550
6.80000	0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00001	0.09550
7.00000	0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
7.20000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
7.40000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
7.60000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
7.80000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
8.00000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
8.20000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
8.40000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
8.60000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
8.80000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
9.00000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
9.20000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
9.40000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
9.60000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
9.80000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550
10.00000	-0.00000	-0.00000	0.00000	-0.04268	0.00000	-0.00000	0.00000	0.09550

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_6G_3$ $(m_1\delta_1)$	${}_6G'_3$	${}_6G''_3$	${}_6I_3$	${}_7G_3$ (m_2)	${}_7G'_3$	${}_7G''_3$	${}_7I_3$
0.	0.	0.04217	-0.20808	0.	0.	0.48660	-1.40043	0.
0.01000	0.00041	0.04009	-0.20809	0.00000	0.00480	0.47263	-1.39368	0.00002
0.02000	0.00080	0.03801	-0.20812	0.00001	0.00945	0.45873	-1.38695	0.00010
0.03000	0.00117	0.03593	-0.20816	0.00002	0.01397	0.44489	-1.38025	0.00021
0.04000	0.00152	0.03385	-0.20821	0.00003	0.01835	0.43113	-1.37355	0.00037
0.05000	0.00185	0.03176	-0.20826	0.00005	0.02259	0.41742	-1.36686	0.00058
0.06000	0.00216	0.02968	-0.20832	0.00007	0.02670	0.40379	-1.36016	0.00083
0.07000	0.00244	0.02760	-0.20837	0.00009	0.03067	0.39022	-1.35346	0.00111
0.08000	0.00271	0.02551	-0.20841	0.00012	0.03450	0.37672	-1.34673	0.00144
0.09000	0.00295	0.02343	-0.20845	0.00015	0.03820	0.36329	-1.33998	0.00180
0.10000	0.00318	0.02134	-0.20847	0.00018	0.04177	0.34992	-1.33320	0.00220
0.11000	0.00338	0.01926	-0.20847	0.00021	0.04520	0.33662	-1.32637	0.00264
0.12000	0.00356	0.01718	-0.20845	0.00024	0.04850	0.32339	-1.31951	0.00311
0.13000	0.00372	0.01509	-0.20840	0.00028	0.05167	0.31023	-1.31259	0.00361
0.14000	0.00386	0.01301	-0.20832	0.00032	0.05471	0.29714	-1.30562	0.00414
0.15000	0.00398	0.01093	-0.20821	0.00036	0.05761	0.28412	-1.29858	0.00470
0.20000	0.00427	0.00054	-0.20706	0.00057	0.07021	0.22009	-1.26224	0.00791
0.25000	0.00404	-0.00976	-0.20461	0.00078	0.07965	0.15794	-1.22352	0.01167
0.30000	0.00330	-0.01989	-0.20052	0.00096	0.08604	0.09779	-1.18183	0.01582
0.35000	0.00205	-0.02978	-0.19447	0.00110	0.08947	0.03981	-1.13671	0.02022
0.40000	0.00032	-0.03931	-0.18624	0.00116	0.09006	-0.01582	-1.08778	0.02472
0.45000	-0.00187	-0.04836	-0.17563	0.00112	0.08793	-0.06890	-1.03478	0.02918
0.50000	-0.00450	-0.05683	-0.16252	0.00096	0.08321	-0.11923	-0.97755	0.03347
0.55000	-0.00754	-0.06457	-0.14687	0.00066	0.07606	-0.16658	-0.91605	0.03747
0.60000	-0.01095	-0.07147	-0.12870	0.00020	0.06661	-0.21076	-0.85034	0.04104
0.65000	-0.01467	-0.07740	-0.10810	-0.00044	0.05504	-0.25155	-0.78059	0.04409
0.70000	-0.01867	-0.08224	-0.08524	-0.00127	0.04151	-0.28876	-0.70707	0.04651
0.80000	-0.02724	-0.08825	-0.03379	-0.00356	0.00936	-0.35173	-0.55034	0.04911
0.90000	-0.03614	-0.08882	0.02295	-0.00673	-0.02829	-0.39851	-0.38418	0.04820
1.00000	-0.04481	-0.08360	0.08144	-0.01078	-0.06978	-0.42842	-0.21381	0.04332
1.10000	-0.05266	-0.07261	0.13760	-0.01566	-0.11341	-0.44133	-0.04516	0.03417
1.20000	-0.05915	-0.05630	0.18723	-0.02126	-0.15749	-0.43772	0.11557	0.02063
1.40000	-0.06615	-0.01146	0.25220	-0.03394	-0.24077	-0.38588	0.39003	-0.01937
1.60000	-0.06326	0.04047	0.25623	-0.04706	-0.30876	-0.28790	0.57227	-0.07466
1.80000	-0.05033	0.08689	0.19886	-0.05857	-0.35422	-0.16429	0.64555	-0.14137
2.00000	-0.02959	0.11716	0.09905	-0.06667	-0.37421	-0.03656	0.61625	-0.21464
2.20000	-0.00493	0.12569	-0.01309	-0.07015	-0.36980	0.07699	0.50894	-0.28942
2.40000	0.01927	0.11309	-0.10804	-0.06867	-0.34519	0.16407	0.35742	-0.36121
2.60000	0.03927	0.08496	-0.16631	-0.06272	-0.30632	0.21927	0.19536	-0.42654
2.80000	0.05276	0.04940	-0.18242	-0.05340	-0.25957	0.24332	0.04952	-0.48321
3.00000	0.05907	0.01437	-0.16297	-0.04210	-0.21073	0.24129	-0.06355	-0.53023
3.20000	0.05893	-0.01431	-0.12146	-0.03021	-0.16431	0.22052	-0.13758	-0.56767
3.40000	0.05397	-0.03372	-0.07248	-0.01885	-0.12326	0.18874	-0.17451	-0.59632
3.60000	0.04609	-0.04359	-0.02760	-0.00881	-0.08909	0.15272	-0.18146	-0.61743
3.80000	0.03707	-0.04549	0.00645	-0.00049	-0.06211	0.11753	-0.16775	-0.63243
4.00000	0.02826	-0.04187	0.02762	0.00603	-0.04180	0.08639	-0.14246	-0.64272
4.20000	0.02052	-0.03522	0.03719	0.01088	-0.02718	0.06082	-0.11302	-0.64953
4.40000	0.01424	-0.02757	0.03816	0.01433	-0.01708	0.04110	-0.08460	-0.65389
4.60000	0.00946	-0.02030	0.03390	0.01668	-0.01038	0.02671	-0.06012	-0.65659
4.80000	0.00604	-0.01416	0.02730	0.01821	-0.00611	0.01671	-0.04074	-0.65821
5.00000	0.00370	-0.00941	0.02036	0.01917	-0.00347	0.01008	-0.02640	-0.65914
5.20000	0.00219	-0.00596	0.01424	0.01975	-0.00191	0.00586	-0.01640	-0.65967
5.40000	0.00124	-0.00362	0.00941	0.02008	-0.00102	0.00329	-0.00979	-0.65995
5.60000	0.00068	-0.00211	0.00591	0.02027	-0.00053	0.00178	-0.00561	-0.66010
5.80000	0.00036	-0.00118	0.00354	0.02037	-0.00026	0.00093	-0.00310	-0.66018
6.00000	0.00018	-0.00064	0.00203	0.02042	-0.00013	0.00047	-0.00165	-0.66022
6.20000	0.00009	-0.00033	0.00111	0.02045	-0.00006	0.00023	-0.00085	-0.66024
6.40000	0.00004	-0.00017	0.00059	0.02046	-0.00003	0.00011	-0.00042	-0.66024
6.60000	0.00002	-0.00008	0.00030	0.02047	-0.00001	0.00005	-0.00020	-0.66025
6.80000	0.00001	-0.00004	0.00014	0.02047	-0.00001	0.00002	-0.00009	-0.66025
7.00000	0.00000	-0.00002	0.00007	0.02047	-0.00000	0.00001	-0.00004	-0.66025
7.20000	0.00000	-0.00001	0.00003	0.02047	-0.00000	0.00000	-0.00002	-0.66025
7.40000	0.00000	-0.00000	0.00001	0.02047	-0.00000	0.00000	-0.00001	-0.66025
7.60000	0.00000	-0.00000	0.00001	0.02047	-0.00000	0.00000	-0.00000	-0.66025
7.80000	0.00000	-0.00000	0.00000	0.02047	-0.00000	0.00000	-0.00000	-0.66025
8.00000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
8.20000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
8.40000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
8.60000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
8.80000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
9.00000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
9.20000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
9.40000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
9.60000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
9.80000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025
10.00000	-0.00000	-0.00000	0.00000	0.02047	0.00000	0.00000	-0.00000	-0.66025

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_8G_3$ (δ_2)	${}_8G_3'$	${}_8G_3''$	${}_1G_4$ (m_3)	${}_1G_4'$	${}_1G_4''$	${}_1I_4$
0.	0.	-0.06296	0.	0.	0.76428	-2.80085	0.
0.01000	-0.00063	-0.06296	0.00002	0.00750	0.73641	-2.77388	0.00004
0.02000	-0.00126	-0.06296	0.00006	0.01473	0.70880	-2.74708	0.00015
0.03000	-0.00189	-0.06296	0.00014	0.02168	0.68147	-2.72042	0.00033
0.04000	-0.00252	-0.06296	0.00025	0.02836	0.65440	-2.69391	0.00058
0.05000	-0.00315	-0.06296	0.00038	0.03477	0.62759	-2.66752	0.00090
0.06000	-0.00378	-0.06295	0.00055	0.04091	0.60104	-2.64124	0.00128
0.07000	-0.00441	-0.06295	0.00074	0.04679	0.57476	-2.61506	0.00172
0.08000	-0.00504	-0.06294	0.00097	0.05241	0.54874	-2.58896	0.00221
0.09000	-0.00567	-0.06293	0.00122	0.05777	0.52298	-2.56294	0.00276
0.10000	-0.00630	-0.06291	0.00149	0.06287	0.49748	-2.53698	0.00337
0.11000	-0.00692	-0.06290	0.00179	0.06772	0.47224	-2.51106	0.00402
0.12000	-0.00755	-0.06288	0.00212	0.07231	0.44726	-2.48519	0.00472
0.13000	-0.00818	-0.06286	0.00248	0.07666	0.42254	-2.45934	0.00546
0.14000	-0.00881	-0.06283	0.00285	0.08077	0.39808	-2.43351	0.00625
0.15000	-0.00944	-0.06280	0.00326	0.08463	0.37387	-2.40768	0.00708
0.20000	-0.01257	-0.06258	0.00561	0.10036	0.25672	-2.27837	0.01173
0.25000	-0.01569	-0.06223	0.00849	0.11041	0.14605	-2.14804	0.01702
0.30000	-0.01879	-0.06172	0.01182	0.11508	0.04194	-2.01592	0.02268
0.35000	-0.02186	-0.06104	0.01554	0.11471	-0.05550	-1.88150	0.02844
0.40000	-0.02489	-0.06016	0.01959	0.10964	-0.14616	-1.74447	0.03407
0.45000	-0.02788	-0.05908	0.02390	0.10021	-0.22991	-1.60475	0.03934
0.50000	-0.03080	-0.05777	0.02839	0.08677	-0.30660	-1.46244	0.04403
0.55000	-0.03365	-0.05624	0.03300	0.06967	-0.37611	-1.31782	0.04795
0.60000	-0.03642	-0.05447	0.03766	0.04928	-0.43835	-1.17135	0.05094
0.65000	-0.03909	-0.05247	0.04231	0.02596	-0.49322	-1.02361	0.05283
0.70000	-0.04166	-0.05024	0.04688	0.00008	-0.54070	-0.87531	0.05349
0.80000	-0.04644	-0.04511	0.05552	-0.05787	-0.61344	-0.58039	0.05066
0.90000	-0.05066	-0.03917	0.06311	-0.12164	-0.65705	-0.29383	0.04172
1.00000	-0.05425	-0.03254	0.06925	-0.18835	-0.67275	-0.02338	0.02624
1.10000	-0.05715	-0.02538	0.07362	-0.25532	-0.66252	0.22351	0.00404
1.20000	-0.05931	-0.01788	0.07601	-0.32008	-0.62905	0.44028	-0.02475
1.40000	-0.06136	-0.00269	0.07452	-0.43467	-0.50600	0.76443	-0.10064
1.60000	-0.06046	0.01140	0.06524	-0.51922	-0.33399	0.92885	-0.19661
1.80000	-0.05697	0.02302	0.05012	-0.56712	-0.14463	0.94137	-0.30587
2.00000	-0.05148	0.03125	0.03196	-0.57778	0.03439	0.83177	-0.42096
2.20000	-0.04472	0.03579	0.01370	-0.55543	0.18272	0.64201	-0.53477
2.40000	-0.03739	0.03689	-0.00223	-0.50752	0.28882	0.41647	-0.64142
2.60000	-0.03015	0.03516	-0.01430	-0.44294	0.34956	0.19425	-0.73667
2.80000	-0.02346	0.03146	-0.02195	-0.37048	0.36869	0.00422	-0.81808
3.00000	-0.01764	0.02666	-0.02542	-0.29769	0.35452	-0.13709	-0.88485
3.20000	-0.01282	0.02152	-0.02551	-0.23020	0.31744	-0.22494	-0.93751
3.40000	-0.00901	0.01661	-0.02327	-0.17155	0.26781	-0.26396	-0.97752
3.60000	-0.00613	0.01230	-0.01973	-0.12332	0.21442	-0.26455	-1.00683
3.80000	-0.00404	0.00875	-0.01573	-0.08559	0.16371	-0.23937	-1.02755
4.00000	-0.00258	0.00600	-0.01190	-0.05739	0.11958	-0.20049	-1.04170
4.20000	-0.00160	0.00396	-0.00857	-0.03720	0.08378	-0.15754	-1.05104
4.40000	-0.00096	0.00252	-0.00591	-0.02332	0.05640	-0.11709	-1.05700
4.60000	-0.00056	0.00155	-0.00390	-0.01414	0.03653	-0.08277	-1.06068
4.80000	-0.00031	0.00092	-0.00248	-0.00830	0.02279	-0.05585	-1.06288
5.00000	-0.00017	0.00053	-0.00151	-0.00471	0.01371	-0.03607	-1.06415
5.20000	-0.00009	0.00029	-0.00089	-0.00259	0.00796	-0.02235	-1.06486
5.40000	-0.00005	0.00016	-0.00050	-0.00138	0.00446	-0.01330	-1.06525
5.60000	-0.00002	0.00008	-0.00028	-0.00071	0.00241	-0.00762	-1.06545
5.80000	-0.00001	0.00004	-0.00015	-0.00036	0.00126	-0.00420	-1.06556
6.00000	-0.00001	0.00002	-0.00007	-0.00017	0.00064	-0.00223	-1.06561
6.20000	-0.00000	0.00001	-0.00004	-0.00008	0.00031	-0.00114	-1.06563
6.40000	-0.00000	0.00000	-0.00002	-0.00004	0.00015	-0.00057	-1.06564
6.60000	-0.00000	0.00000	-0.00001	-0.00002	0.00007	-0.00027	-1.06565
6.80000	-0.00000	0.00000	-0.00000	-0.00001	0.00003	-0.00012	-1.06565
7.00000	-0.00000	0.00000	-0.00000	-0.00000	0.00001	-0.00006	-1.06565
7.20000	-0.00000	0.00000	-0.00000	-0.00000	0.00001	-0.00002	-1.06565
7.40000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00001	-1.06565
7.60000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
7.80000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
8.00000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
8.20000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
8.40000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
8.60000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
8.80000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
9.00000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
9.20000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
9.40000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
9.60000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
9.80000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565
10.00000	-0.00000	0.00000	-0.00000	-0.00000	0.00000	-0.00000	-1.06565

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	$_2G_4$ $(m_1\ m_2)$	$_2G_4'$	$_2G_4''$	$_2I_4$	$_3G_4$ (δ_3)	$_3G_4'$	$_3G_4''$
0.	0.	-2.94233	12.88045	0.	0.	-0.05650	0.
0.01000	-0.02878	-2.81456	12.67293	-0.00014	-0.00057	-0.05650	0.00002
0.02000	-0.05630	-2.68887	12.46686	-0.00057	-0.00113	-0.05650	0.00006
0.03000	-0.08257	-2.56522	12.26216	-0.00127	-0.00170	-0.05650	0.00015
0.04000	-0.10761	-2.44362	12.05877	-0.00222	-0.00226	-0.05650	0.00026
0.05000	-0.13145	-2.32404	11.85663	-0.00341	-0.00283	-0.05650	0.00040
0.06000	-0.15410	-2.20648	11.65570	-0.00484	-0.00339	-0.05649	0.00057
0.07000	-0.17558	-2.09092	11.45591	-0.00649	-0.00395	-0.05648	0.00077
0.08000	-0.19592	-1.97736	11.25723	-0.00835	-0.00452	-0.05648	0.00099
0.09000	-0.21514	-1.86578	11.05960	-0.01041	-0.00508	-0.05646	0.00125
0.10000	-0.23324	-1.75616	10.86299	-0.01265	-0.00565	-0.05645	0.00153
0.11000	-0.25027	-1.64851	10.66737	-0.01507	-0.00621	-0.05643	0.00184
0.12000	-0.26622	-1.54281	10.47269	-0.01765	-0.00678	-0.05641	0.00217
0.13000	-0.28113	-1.43906	10.27893	-0.02039	-0.00734	-0.05639	0.00253
0.14000	-0.29501	-1.33723	10.08606	-0.02327	-0.00791	-0.05636	0.00291
0.15000	-0.30788	-1.23733	9.89407	-0.02629	-0.00847	-0.05633	0.00331
0.20000	-0.35778	-0.76640	8.94656	-0.04303	-0.01128	-0.05611	0.00566
0.25000	-0.38530	-0.34234	8.01911	-0.06169	-0.01408	-0.05576	0.00850
0.30000	-0.39277	0.03585	7.11207	-0.08122	-0.01685	-0.05525	0.01176
0.35000	-0.38246	0.36923	6.22729	-0.10067	-0.01960	-0.05458	0.01535
0.40000	-0.35658	0.65900	5.36786	-0.11921	-0.02231	-0.05371	0.01922
0.45000	-0.31727	0.90651	4.53789	-0.13611	-0.02497	-0.05265	0.02328
0.50000	-0.26661	1.11336	3.74218	-0.15075	-0.02757	-0.05138	0.02747
0.55000	-0.20658	1.28139	2.98603	-0.16261	-0.03010	-0.04990	0.03172
0.60000	-0.13908	1.41271	2.27492	-0.17128	-0.03256	-0.04821	0.03598
0.65000	-0.06588	1.50972	1.61422	-0.17642	-0.03492	-0.04631	0.04016
0.70000	0.01137	1.57506	1.00898	-0.17780	-0.03718	-0.04420	0.04422
0.80000	0.17210	1.62248	-0.01820	-0.16867	-0.04137	-0.03939	0.05175
0.90000	0.33287	1.58022	-0.78201	-0.14338	-0.04504	-0.03388	0.05817
1.00000	0.48605	1.47510	-1.27601	-0.10235	-0.04813	-0.02780	0.06314
1.10000	0.62668	1.33356	-1.51442	-0.04659	-0.05059	-0.02131	0.06644
1.20000	0.75235	1.17961	-1.53149	0.02249	-0.05238	-0.01458	0.06793
1.40000	0.95966	0.90770	-1.11446	0.19460	-0.05394	-0.00113	0.06537
1.60000	1.12301	0.74618	-0.50980	0.40340	-0.05291	0.01113	0.05622
1.80000	1.26515	0.68767	-0.13865	0.64241	-0.04965	0.02105	0.04236
2.00000	1.40027	0.66163	-0.19469	0.90904	-0.04470	0.02793	0.02629
2.20000	1.52636	0.58514	-0.61483	1.20195	-0.03869	0.03158	0.01047
2.40000	1.62740	0.40655	-1.17142	1.51792	-0.03226	0.03226	-0.00312
2.60000	1.68199	0.12457	-1.61108	1.84980	-0.02595	0.03056	-0.01325
2.80000	1.67311	-0.21851	-1.76467	2.18646	-0.02015	0.02722	-0.01955
3.00000	1.59475	-0.55924	-1.59125	2.51438	-0.01512	0.02298	-0.02228
3.20000	1.45361	-0.83773	-1.16067	2.82015	-0.01097	0.01850	-0.02217
3.40000	1.26647	-1.01499	-0.60244	3.09275	-0.00770	0.01425	-0.02010
3.60000	1.05521	-1.07922	-0.05074	3.32513	-0.00523	0.01053	-0.01698
3.80000	0.84155	-1.04250	0.39447	3.51468	-0.00345	0.00748	-0.01350
4.00000	0.64314	-0.93195	0.68352	3.66278	-0.00220	0.00512	-0.01019
4.20000	0.47153	-0.77977	0.81347	3.77374	-0.00136	0.00337	-0.00733
4.40000	0.33203	-0.61527	0.81328	3.85355	-0.00081	0.00215	-0.00504
4.60000	0.22477	-0.46025	0.72623	3.90871	-0.00047	0.00132	-0.00333
4.80000	0.14642	-0.32768	0.59542	3.94539	-0.00027	0.00078	-0.00211
5.00000	0.09185	-0.22269	0.45503	3.96886	-0.00014	0.00045	-0.00129
5.20000	0.05553	-0.14480	0.32716	3.98334	-0.00008	0.00025	-0.00076
5.40000	0.03237	-0.09025	0.22266	3.99195	-0.00004	0.00013	-0.00043
5.60000	0.01821	-0.05400	0.14408	3.99689	-0.00002	0.00007	-0.00023
5.80000	0.00988	-0.03105	0.08893	3.99962	-0.00001	0.00004	-0.00012
6.00000	0.00518	-0.01718	0.05248	4.00108	-0.00000	0.00002	-0.00006
6.20000	0.00263	-0.00915	0.02967	4.00183	-0.00000	0.00001	-0.00003
6.40000	0.00128	-0.00470	0.01610	4.00221	-0.00000	0.00000	-0.00001
6.60000	0.00061	-0.00233	0.00839	4.00239	-0.00000	0.00000	-0.00001
6.80000	0.00028	-0.00111	0.00420	4.00248	-0.00000	0.00000	-0.00000
7.00000	0.00012	-0.00051	0.00203	4.00251	-0.00000	0.00000	-0.00000
7.20000	0.00005	-0.00023	0.00094	4.00253	-0.00000	0.00000	-0.00000
7.40000	0.00002	-0.00010	0.00042	4.00254	-0.00000	0.00000	-0.00000
7.60000	0.00001	-0.00004	0.00018	4.00254	-0.00000	0.00000	-0.00000
7.80000	0.00000	-0.00002	0.00008	4.00254	-0.00000	0.00000	-0.00000
8.00000	0.00000	-0.00001	0.00003	4.00254	0.00000	0.00000	-0.00000
8.20000	0.00000	-0.00000	0.00001	4.00254	0.00000	0.00000	-0.00000
8.40000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
8.60000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
8.80000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
9.00000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
9.20000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
9.40000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
9.60000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
9.80000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000
10.00000	0.00000	-0.00000	0.00000	4.00254	0.00000	0.00000	-0.00000

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Continued

η	${}_1G_5$ $(m_1 m_3)$	${}_1G_5'$	${}_1G_5''$	${}_1I_5$	${}_1G_6$ $(m_2 m_3)$	${}_1G_6'$	${}_1G_6''$	${}_1I_6$
0.	0.	-5.39703	28.85773	0.	0.	-18.38894	120.38628	0.
0.01000	-0.05254	-5.11162	28.22641	-0.00027	-0.17793	-17.20271	116.87207	-0.00090
0.02000	-0.10225	-4.83248	27.60307	-0.00104	-0.34417	-16.05129	113.42126	-0.00352
0.03000	-0.14921	-4.55953	26.98750	-0.00230	-0.49907	-14.93407	110.03264	-0.00775
0.04000	-0.19346	-4.29270	26.37953	-0.00402	-0.64296	-13.85044	106.70500	-0.01346
0.05000	-0.23508	-4.03192	25.77898	-0.00616	-0.77619	-12.79977	103.43724	-0.02057
0.06000	-0.27412	-3.77710	25.18571	-0.00871	-0.89907	-11.78149	100.22832	-0.02895
0.07000	-0.31064	-3.52818	24.59955	-0.01164	-1.01192	-10.79501	97.07722	-0.03852
0.08000	-0.34471	-3.28508	24.02036	-0.01491	-1.11507	-9.83976	93.98301	-0.04916
0.09000	-0.37636	-3.04775	23.44803	-0.01852	-1.20882	-8.91517	90.94477	-0.06079
0.10000	-0.40568	-2.81610	22.88242	-0.02243	-1.29348	-8.02068	87.96169	-0.07331
0.11000	-0.43271	-2.59008	22.32343	-0.02663	-1.36933	-7.15575	85.03294	-0.08663
0.12000	-0.45750	-2.36961	21.77095	-0.03108	-1.43669	-6.31984	82.15778	-0.10066
0.13000	-0.48012	-2.15464	21.22489	-0.03577	-1.49582	-5.51242	79.33549	-0.11533
0.14000	-0.50061	-1.94509	20.68516	-0.04068	-1.54703	-4.73296	76.56540	-0.13055
0.15000	-0.51904	-1.74091	20.15169	-0.04578	-1.59058	-3.98094	73.84688	-0.14625
0.20000	-0.58198	-0.79835	17.57584	-0.07350	-1.70279	-0.61474	61.00780	-0.22928
0.25000	-0.60096	0.01914	15.14840	-0.10324	-1.66224	2.14003	49.38053	-0.31398
0.30000	-0.58203	0.71891	12.86645	-0.13296	-1.49799	4.34269	38.91635	-0.39345
0.35000	-0.53091	1.30821	10.72984	-0.16091	-1.23622	6.05040	29.57663	-0.46216
0.40000	-0.45293	1.79435	8.74040	-0.18560	-0.90028	7.31854	21.32827	-0.51583
0.45000	-0.35307	2.18475	6.90117	-0.20584	-0.51079	8.20089	14.13985	-0.55130
0.50000	-0.23592	2.48703	5.21579	-0.22062	-0.08575	8.74965	7.97842	-0.56632
0.55000	-0.10570	2.70896	3.68780	-0.22921	0.35945	9.01524	2.80690	-0.55954
0.60000	0.03377	2.85848	2.32016	-0.23104	0.81187	9.04612	-1.41781	-0.53026
0.65000	0.17907	2.94368	1.11471	-0.22574	1.26092	8.88840	-4.74611	-0.47841
0.70000	0.32720	2.97266	0.07184	-0.21309	1.69829	8.58548	-7.23633	-0.40436
0.80000	0.62188	2.89431	-1.53376	-0.16557	2.51492	7.70164	-9.97508	-0.19296
0.90000	0.90174	2.68619	-2.53200	-0.08921	3.23386	6.67279	-10.24684	0.09534
1.00000	1.15673	2.40593	-2.99066	0.01395	3.85181	5.71153	-8.74177	0.45042
1.10000	1.38218	2.10287	-3.00582	0.14115	4.38323	4.95992	-6.17059	0.86280
1.20000	1.57784	1.81570	-2.69323	0.28939	4.85327	4.49056	-3.20154	1.32502
1.40000	1.89395	1.38306	-1.57752	0.63801	5.72449	4.38970	1.79512	2.38311
1.60000	2.14666	1.17928	-0.53088	1.04275	6.65605	4.98231	3.48207	3.61914
1.80000	2.37599	1.12820	-0.10660	1.49518	7.71437	5.53406	1.49987	5.05431
2.00000	2.59852	1.08638	-0.42424	1.99276	8.82481	5.42512	-2.82091	6.70857
2.20000	2.80235	0.92413	-1.25186	2.53338	9.82107	4.37867	-7.55895	8.57665
2.40000	2.95576	0.57846	-2.18788	3.11035	10.51914	2.48387	-11.08802	10.61701
2.60000	3.02258	0.06756	-2.85054	3.70989	10.78083	0.08540	-12.51480	12.75503
2.80000	2.97709	-0.52773	-3.00977	4.31185	10.54942	-2.37286	-11.72366	14.89627
3.00000	2.81303	-1.10025	-2.63321	4.89277	9.85493	-4.48693	-9.18577	16.94377
3.20000	2.54500	-1.55393	-1.85332	5.43009	8.79625	-5.98245	-5.67800	18.81388
3.40000	2.20346	-1.82925	-0.88845	5.90586	7.51095	-6.74803	-2.01275	20.44715
3.60000	1.82624	-1.91175	0.04199	6.30910	6.14345	-6.82098	1.16024	21.81282
3.80000	1.45005	-1.82561	0.77812	6.63644	4.81939	-6.34312	3.45474	22.90750
4.00000	1.10407	-1.61870	1.24393	6.89116	3.63017	-5.50578	4.75613	23.74966
4.20000	0.80694	-1.34612	1.44025	7.08135	2.62819	-4.50055	5.16335	24.37213
4.40000	0.56669	-1.05712	1.41951	7.21774	1.83051	-3.48520	4.90025	24.81461
4.60000	0.38274	-0.78780	1.25638	7.31179	1.22748	-2.56762	4.22854	25.11735
4.80000	0.24882	-0.55916	1.02380	7.37418	0.79302	-1.80520	3.38341	25.31686
5.00000	0.15582	-0.37905	0.77890	7.41405	0.49392	-1.21407	2.53884	25.44358
5.20000	0.09406	-0.24595	0.55808	7.43859	0.29672	-0.78251	1.79962	25.52121
5.40000	0.05476	-0.15302	0.37879	7.45316	0.17203	-0.48408	1.21102	25.56709
5.60000	0.03076	-0.09141	0.24456	7.46151	0.09629	-0.28778	0.77643	25.59327
5.80000	0.01668	-0.05250	0.15067	7.46613	0.05205	-0.16458	0.47556	25.60769
6.00000	0.00874	-0.02901	0.08878	7.46859	0.02719	-0.09062	0.27885	25.61537
6.20000	0.00442	-0.01544	0.05013	7.46986	0.01372	-0.04807	0.15679	25.61932
6.40000	0.00216	-0.00792	0.02716	7.47050	0.00670	-0.02459	0.08465	25.62128
6.60000	0.00102	-0.00392	0.01414	7.47080	0.00316	-0.01213	0.04393	25.62223
6.80000	0.00047	-0.00187	0.00708	7.47094	0.00144	-0.00578	0.02194	25.62267
7.00000	0.00021	-0.00086	0.00341	7.47101	0.00064	-0.00265	0.01055	25.62287
7.20000	0.00009	-0.00038	0.00158	7.47104	0.00027	-0.00118	0.00489	25.62295
7.40000	0.00004	-0.00016	0.00071	7.47105	0.00011	-0.00051	0.00218	25.62299
7.60000	0.00002	-0.00007	0.00031	7.47105	0.00005	-0.00021	0.00094	25.62301
7.80000	0.00001	-0.00003	0.00013	7.47106	0.00002	-0.00008	0.00039	25.62301
8.00000	0.00000	-0.00001	0.00005	7.47106	0.00001	-0.00003	0.00016	25.62301
8.20000	0.00000	-0.00000	0.00002	7.47106	0.00000	-0.00001	0.00006	25.62302
8.40000	0.00000	-0.00000	0.00001	7.47106	0.00000	-0.00000	0.00002	25.62302
8.60000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00001	25.62302
8.80000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302
9.00000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302
9.20000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302
9.40000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302
9.60000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302
9.80000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302
10.00000	0.00000	-0.00000	0.00000	7.47106	0.00000	-0.00000	0.00000	25.62302

TABLE I.- AXISYMMETRIC STAGNATION-POINT UNIVERSAL FUNCTIONS

$\sigma_o = 0.75$ - Concluded

η	${}_1G_7$ $(m_1 m_2 m_3)$	${}_1G'_7$	${}_1G''_7$	${}_1I_7$	${}_2G_7$ $(\delta_1 \delta_2 \delta_3)$	${}_2G'_7$	${}_2G''_7$
0.	0.	311.53850	-2649.20850	0.	0.	-0.06284	0.
0.01000	2.98477	285.59945	-2539.12589	0.01514	-0.00063	-0.06284	-0.00001
0.02000	5.71561	260.74561	-2432.15500	0.05885	-0.00126	-0.06284	-0.00005
0.03000	8.20320	236.94621	-2328.22498	0.12864	-0.00189	-0.06284	-0.00010
0.04000	10.45794	214.17120	-2227.26718	0.22214	-0.00251	-0.06284	-0.00018
0.05000	12.48994	192.39118	-2129.21548	0.33706	-0.00314	-0.06284	-0.00029
0.06000	14.30899	171.57741	-2034.00645	0.47123	-0.00377	-0.06284	-0.00041
0.07000	15.92461	151.70178	-1941.57809	0.62256	-0.00440	-0.06285	-0.00056
0.08000	17.34606	132.73678	-1851.87093	0.78907	-0.00503	-0.06286	-0.00074
0.09000	18.58230	114.65549	-1764.82709	0.96886	-0.00566	-0.06286	-0.00093
0.10000	19.64203	97.43155	-1680.39084	1.16013	-0.00628	-0.06287	-0.00115
0.11000	20.53370	81.03916	-1598.50772	1.36114	-0.00691	-0.06289	-0.00139
0.12000	21.26550	65.45306	-1519.12506	1.57027	-0.00754	-0.06290	-0.00165
0.13000	21.84537	50.64850	-1442.19167	1.78595	-0.00817	-0.06292	-0.00194
0.14000	22.28099	36.60123	-1367.65779	2.00670	-0.00880	-0.06294	-0.00224
0.15000	22.57983	23.28751	-1295.47491	2.23111	-0.00943	-0.06297	-0.00257
0.20000	22.26695	-33.07637	-968.20860	3.36401	-0.01258	-0.06314	-0.00454
0.25000	19.52270	-74.40167	-693.05798	4.41736	-0.01575	-0.06343	-0.00702
0.30000	15.03599	-103.16685	-464.99988	5.28731	-0.01893	-0.06385	-0.00996
0.35000	9.37794	-121.60851	-279.36093	5.90149	-0.02213	-0.06443	-0.01328
0.40000	3.01359	-131.73587	-131.70312	6.21338	-0.02537	-0.06518	-0.01690
0.45000	-3.68705	-135.34049	-17.74950	6.19729	-0.02866	-0.06612	-0.02071
0.50000	-10.43821	-134.00325	66.65483	5.84388	-0.03199	-0.06726	-0.02459
0.55000	-17.02805	-129.10005	125.55276	5.15619	-0.03538	-0.06858	-0.02838
0.60000	-23.30850	-121.80770	162.86196	4.14626	-0.03885	-0.07009	-0.03196
0.65000	-29.18551	-113.11085	182.34447	2.83209	-0.04240	-0.07177	-0.03515
0.70000	-34.60963	-103.81082	187.56501	1.23527	-0.04603	-0.07360	-0.03780
0.80000	-44.07163	-85.75809	168.21091	-2.71388	-0.05359	-0.07756	-0.04084
0.90000	-51.86897	-70.86981	127.62624	-7.52333	-0.06155	-0.08163	-0.03993
1.00000	-58.39325	-60.36488	82.86979	-13.04519	-0.06990	-0.08538	-0.03419
1.10000	-64.08304	-54.06687	44.78627	-19.17424	-0.07859	-0.08829	-0.02316
1.20000	-69.31500	-51.01190	18.41802	-25.84667	-0.08751	-0.08984	-0.00690
1.40000	-79.34047	-49.86045	-0.58790	-40.71563	-0.10534	-0.08692	0.03846
1.60000	-89.31551	-49.76606	2.67105	-57.58147	-0.12160	-0.07393	0.09160
1.80000	-99.19720	-49.03475	3.08622	-76.43528	-0.13422	-0.05072	0.13824
2.00000	-108.98354	-49.07400	-4.05063	-97.25326	-0.14138	-0.01994	0.16555
2.20000	-118.91857	-50.38786	-7.16316	-120.03897	-0.14200	0.01375	0.16676
2.40000	-129.08226	-50.79470	6.51844	-144.83746	-0.13604	0.04508	0.14285
2.60000	-138.91778	-46.44576	39.79996	-171.65177	-0.12442	0.06969	0.10113
2.80000	-147.12461	-34.14824	83.76086	-200.29696	-0.10879	0.08504	0.05202
3.00000	-151.99639	-13.24723	123.18920	-230.27887	-0.09106	0.09068	0.00547
3.20000	-152.00318	13.88434	144.21997	-260.76953	-0.07308	0.08788	-0.03143
3.40000	-146.32706	42.73295	139.93439	-290.69901	-0.05631	0.07898	-0.05538
3.60000	-135.13359	68.26512	111.98080	-318.93040	-0.04172	0.06659	-0.06647
3.80000	-119.51282	86.48970	68.58332	-344.45590	-0.02975	0.05309	-0.06709
4.00000	-101.16353	95.40084	20.67978	-366.55323	-0.02044	0.04023	-0.06062
4.20000	-81.96857	95.12690	-21.86837	-384.86543	-0.01354	0.02909	-0.05043
4.40000	-63.60855	87.44459	-52.69460	-399.39738	-0.00866	0.02012	-0.03920
4.60000	-47.30932	74.98499	-69.59960	-410.44748	-0.00535	0.01335	-0.02874
4.80000	-33.75151	60.45381	-73.83691	-418.50500	-0.00319	0.00851	-0.02000
5.00000	-23.11585	46.07602	-68.70586	-424.14372	-0.00184	0.00522	-0.01326
5.20000	-15.21030	33.33455	-58.11129	-427.93383	-0.00103	0.00308	-0.00840
5.40000	-9.62268	22.96317	-45.50699	-430.38255	-0.00055	0.00175	-0.00510
5.60000	-5.85705	15.09889	-33.35863	-431.90432	-0.00029	0.00096	-0.00297
5.80000	-3.43206	9.49476	-23.05466	-432.81458	-0.00015	0.00051	-0.00166
6.00000	-1.93717	5.71930	-15.09720	-433.33894	-0.00007	0.00026	-0.00090
6.20000	-1.05376	3.30444	-9.40170	-433.63001	-0.00003	0.00013	-0.00047
6.40000	-0.55269	1.83330	-5.58334	-433.78577	-0.00002	0.00006	-0.00023
6.60000	-0.27962	0.97761	-3.16887	-433.86616	-0.00001	0.00003	-0.00011
6.80000	-0.13652	0.50147	-1.72184	-433.90620	-0.00000	0.00001	-0.00005
7.00000	-0.06434	0.24762	-0.89698	-433.92544	-0.00000	0.00001	-0.00002
7.20000	-0.02929	0.11778	-0.44852	-433.93438	-0.00000	0.00000	-0.00001
7.40000	-0.01288	0.05399	-0.21550	-433.93837	-0.00000	0.00000	-0.00000
7.60000	-0.00548	0.02386	-0.09957	-433.94012	-0.00000	0.00000	-0.00000
7.80000	-0.00226	0.01017	-0.04427	-433.94084	-0.00000	0.00000	-0.00000
8.00000	-0.00091	0.00419	-0.01896	-433.94114	-0.00000	0.00000	-0.00000
8.20000	-0.00036	0.00166	-0.00782	-433.94126	-0.00000	0.00000	-0.00000
8.40000	-0.00015	0.00064	-0.00311	-433.94131	-0.00000	0.00000	-0.00000
8.60000	-0.00007	0.00024	-0.00119	-433.94133	-0.00000	0.00000	-0.00000
8.80000	-0.00004	0.00008	-0.00044	-433.94134	-0.00000	0.00000	-0.00000
9.00000	-0.00003	0.00003	-0.00016	-433.94135	-0.00000	0.00000	-0.00000
9.20000	-0.00002	0.00001	-0.00005	-433.94135	-0.00000	0.00000	-0.00000
9.40000	-0.00002	0.00000	-0.00002	-433.94135	-0.00000	0.00000	-0.00000
9.60000	-0.00002	0.00000	-0.00001	-433.94136	-0.00000	0.00000	-0.00000
9.80000	-0.00002	0.00000	-0.00000	-433.94136	-0.00000	0.00000	-0.00000
10.00000	-0.00002	0.00000	-0.00000	-433.94136	-0.00000	0.00000	-0.00000

2/17/25

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